The cobordism hypothesis—after Baez–Dolan, Costello, Hopkins–Lurie, and Lurie—asserts that for a symmetric monoidal \((\infty, n)\)-category \(C\) in which every object has a dual and every \(k\)-morphism has a left and right adjoint for \(k < n\), there is an equivalence \(\text{TQFT}(C) = \text{obj}(C)\) between \(C\)-valued framed topological field theories and objects of \(C\). This is the formulation due to Lurie. I’ll give a proof of the cobordism hypothesis based on factorization homology.

Factorization homology is a multiplicative analogue of ordinary homology. Usual homology integrates an abelian group, chain complex, or spectrum over a manifold \(M\), which one can think as the moduli space of points in \(M\) itself. The result takes disjoint unions of manifolds to direct sums. The alpha version of factorization homology integrates an \(E_n\)-algebra over a moduli space of finite subsets of a manifold \(M\). The beta version of factorization homology integrates an \((\infty, n)\)-category over the moduli space of stratifications of \(M\). The result takes disjoint unions to tensor products. I’ll define this beta version of factorization homology. It satisfies a version of the Eilenberg–Steenrod axioms—this part is work in progress. These Eilenberg–Steenrod axioms together with an argument in the spirit of Pontryagin–Thom theory implies the cobordism hypothesis. This is joint work with David Ayala.