In Morse theory, it’s important to understand how a gradient trajectory can break into multiple gradient trajectories—for instance, this is how one proves the Morse chain complex is a chain complex. In this talk, I’ll define a stack classifying families of domains of such broken and breaking trajectories. I’ll then sketch a proof that the $\infty$-category of factorizable sheaves on this stack is equivalent to the $\infty$-category of non-unital $A_\infty$ algebras. Much of this is motivated by a strategy to formulate Morse theory and Floer theory as a deformation problem (similar to Cohen-Jones-Segal), but I may not talk too much about that because the details are not worked out yet. This is joint work with Jacob Lurie.