Let $M$ be a manifold, and let $\text{Mod}(M)$ be the group of diffeomorphisms of $M$ modulo isotopy (the mapping class group). The Nielsen realization problem for diffeomorphisms asks, “Can a given subgroup $G < \text{Mod}(M)$ be lifted to the diffeomorphism group $\text{Diff}(M)$?” This question about group actions is related to a question about flat connections on fiber bundles with fiber $M$. In the case $M$ is a closed surface, the answer is “yes” for finite $G$ (by work of Kerckhoff) and “no” for $G = \text{Mod}(M)$ (by work of Morita). For most infinite $G < \text{Mod}(M)$, we don’t know. I will discuss some obstructions that can be used to show that certain groups don’t lift. Some of this work is joint with Nick Salter.