Topology Seminar

Inna Zakharevich

of University of Chicago will be speaking on

The annihilator of the Lefschetz motive

on March 7 at 4:30 in
MIT Room 2-131

The Grothendieck ring of varieties is defined to be the free abelian group generated by varieties, modulo the relation that for a closed subvariety Y of X, $[X] = [Y] + [X \backslash Y]$; the ring structure is defined via the Cartesian product. For example, if X and Y are piecewise isomorphic, in the sense that there exist stratifications on X and Y with isomorphic strata, then $[X] = [Y]$ in the Grothendieck ring.

There are two important questions about this ring:

1. What does it mean when two varieties X and Y have equal classes in the Grothendieck ring? Must X and Y be piecewise isomorphic?

2. Is the class of the affine line a zero divisor?

Last December Borisov answered both of these questions with a single example, by constructing an element $[X] - [Y]$ in the kernel of multiplication by the affine line; in a beautiful coincidence, it turned out that $X \times{}^1$ and $Y \times{}^1$ were not piecewise isomorphic. In this talk we will describe an approach using algebraic K-theory to construct a topological version of the Grothendieck ring of varieties. We shall prove that π_1 of this space is generated by birational automorphisms of varieties which extend to piecewise automorphisms, which allows us to construct a group that surjects onto the kernel of multiplication by the affine line. By analyzing this group we will sketch a proof that Borisov's coincidence was not a coincidence at all: any element in the kernel of multiplication by the affine line can be represented as $[X] - [Y]$, where $X \times{}^1$ and $Y \times{}^1$ are not piecewise isomorphic.