I will describe certain surprising features of algebraic geometry that arise if one works exclusively with perfect rings of positive characteristic $p$; these features are strongly reminiscent of derived algebraic geometry. When combined with some higher algebraic $K$-theory, this will allow us to attach “determinants” to certain mildly non-linear objects. Time permitting, I will explain why these determinants are useful in constructing an object of interest in arithmetic geometry: an algebraic variety in characteristic $p$ that parametrizes $\mathbb{Z}_p$-lattices in a finite dimensional $\mathbb{Q}_p$-vector space. This is joint work with Peter Scholze.