A saturated fusion system associated to a finite group $G$ encodes the $p$-structure of the group as the Sylow $p$-subgroup enriched with additional conjugation. The fusion system contains just the right amount of algebraic information to for instance reconstruct the $p$-completion of $BG$, but not $BG$ itself. Abstract saturated fusion systems $F$ without ambient groups exist, and these have ($p$-completed) classifying spaces $BF$ as well. In spectra, the suspension spectrum of $BF$ becomes a retract of the suspension spectrum of $BS$, for the Sylow $p$-subgroup $S$, so $BF$ gets encoded as a characteristic idempotent in the double Burnside ring of $S$. This way of looking as fusion systems as stable retracts of their Sylow $p$-subgroups is a very useful tool for generalizing theorems from groups or $p$-groups to saturated fusion systems. In joint work with Tomer Schlank and Nat Stapleton, we use this retract approach to do Hopkins-Kuhn-Ravenel character theory for all saturated fusion systems by building on the theorems for finite $p$-groups.