Guozhen Wang of MIT will be speaking on

Unstable Chromatic Homotopy Theory

on April 14 at 4:30 in
MIT Room 2-131

Using the self maps provided by the Hopkins-Smith periodicity theorem, we can decompose the unstable homotopy groups of a space into its periodic parts, except some lower stems. For fixed n, using the Bousfield-Kuhn functor we can associate to any space a spectrum, which captures the \( v_n \)-periodic part of its homotopy groups.

I will talk about the homotopy type of the Bousfield-Kuhn functor applied to spheres, which would tell us much about the \( v_n \)-periodic part of the homotopy groups of spheres provided we have a good understanding of the telescope conjecture. I will make use of the Goodwillie tower of the identity functor, which resolves the unstable spheres into spectra which are the Steinberg summands of the classifying spaces of the additive groups of vector spaces over finite fields.

By understanding the attaching maps of the Goodwillie tower after applying the Bousfield-Kuhn functor, we would be able to determine the homotopy type of its effect on spheres. As an example of how this works in concrete computations, I will compute the K(2)-local homotopy groups of the three sphere at primes \( p > 3 \).

The computations show that the unstable homotopy groups not only have finite \( p \)-torsion, their K(2)-local parts also have finite \( v_1 \)-torsion, which indicates there might be a more general finite \( v_n \)-torsion phenomena in the unstable world, conjectured by many people.