Consider a flavor of structured ring spectra that can be described as algebras over an operad $O$ in spectra. A natural question to ask is when the fundamental adjunction comparing $O$-algebra spectra with coalgebra spectra over the associated Koszul dual comonad $K$ can be modified to turn it into an equivalence of homotopy theories. In a paper published in 2012, Francis and Gaitsgory conjecture that replacing $O$-algebras with the full subcategory of homotopy pro-nilpotent $O$-algebras will do the trick. In joint work with Kathryn Hess we show that every 0-connected $O$-algebra is homotopy pro-nilpotent; i.e. is the homotopy limit of a tower of nilpotent $O$-algebras.

This talk will describe recent work, joint with Michael Ching, that resolves in the affirmative the 0-connected case of the Francis-Gaitsgory conjecture; that replacing $O$-algebras with 0-connected $O$-algebras turns the fundamental adjunction into an equivalence of homotopy theories. This can be thought of as a spectral algebra analog of the fundamental work of Quillen and Sullivan on the rational homotopy theory of spaces, the subsequent work of Goerss and Mandell on the $p$-adic homotopy theory of spaces, and the work of Mandell on integral cochains and homotopy type. Corollaries include the following: (i) 0-connected $O$-algebra spectra are weakly equivalent if and only if their $TQ$-homology spectra are weakly equivalent as derived $K$-coalgebras, and (ii) if a $K$-coalgebra spectrum is 0-connected and cofibrant, then it comes from the $TQ$-homology spectrum of an $O$-algebra.