Topology Seminar

John Harper

of Ohio State University will be speaking on

Derived Koszul duality and TQ-homology completion

on February 17 at 4:30 in
MIT Room 2-131

Consider a flavor of structured ring spectra that can be described as algebras over an operad O in spectra. A natural question to ask is when the fundamental adjunction comparing O-algebra spectra with coalgebra spectra over the associated Koszul dual comonad K can be modified to turn it into an equivalence of homotopy theories. In a paper published in 2012, Francis and Gaitsgory conjecture that replacing O-algebras with the full subcategory of homotopy pro-nilpotent O-algebras will do the trick. In joint work with Kathryn Hess we show that every 0-connected O-algebra is homotopy pro-nilpotent; i.e. is the homotopy limit of a tower of nilpotent O-algebras.

This talk will describe recent work, joint with Michael Ching, that resolves in the affirmative the 0-connected case of the Francis-Gaitsgory conjecture; that replacing O-algebras with 0-connected O-algebras turns the fundamental adjunction into an equivalence of homotopy theories. This can be thought of as a spectral algebra analog of the fundamental work of Quillen and Sullivan on the rational homotopy theory of spaces, the subsequent work of Goerss and Mandell on the p-adic homotopy theory of spaces, and the work of Mandell on integral cochains and homotopy type. Corollaries include the following: (i) 0-connected O-algebra spectra are weakly equivalent if and only if their TQ-homology spectra are weakly equivalent as derived K-coalgebras, and (ii) if a K-coalgebra spectrum is 0-connected and cofibrant, then it comes from the TQ-homology spectrum of an O-algebra.