We present a new model category structure on the category of chain complexes over a ring $R$, called the $n$-projective model structure, whose cofibrant objects are given by the class of chain complexes with projective dimension at most $n$ (or $n$-projective complexes). One interesting application of this structure consists in finding another way to compute extension groups $\text{Ext}_{R}^{i}(M, N)$ for every pair of left $R$-modules $M$ and $N$, by using certain cofibrant and fibrant replacements of the sphere chain complexes $S^{0}(M)$ and $S^{i}(N)$, respectively. Recall that one normally computes $\text{Ext}_{R}^{i}(M, N)$ by using either a left resolution of $M$ by projective modules or a right resolution of $N$ by injective modules. Somewhat surprisingly, there turn out to be many other ways to do it. We prove that one can use a left resolution of $M$ by modules of projective dimension at most $n$. The disadvantage of doing so is that we use right resolutions of $N$ by a class of modules which is hard to describe.