Much like for vector bundles, one can attempt to study bundles with fiber a manifold using characteristic classes, which are invariants that correspond to elements of the cohomology ring of the classifying space $\text{BDiff } M$. The easiest of these to define are the so-called “generalized Miller-Morita-Mumford classes”. In the case when the manifold $M = S_g$ is a surface, the Madsen-Weiss theorem together with Harer stability imply that as $g$ grows, a large number of these classes become non-zero. On the other hand, relationship between $\text{BDiff } S_g$ and the moduli space of Riemann surfaces (which has finite dimension) implies that a large number of these classes are zero.

Recently, Galatius and Randall-Williams proved an analogue of the Madsen-Weiss theorem and of Harer stability for the case when $M$ is a connected sum of products of spheres $S^d \times S^d$. I will describe the implications of their results on the study of generalized MMM-classes, other vanishing and non-vanishing results about the MMM-classes for such manifolds, and whether $\text{BDiff } M$ could be modeled by a finite-dimensional space.