Atiyah-Segal and others define a twisted form of $K$-theory associated to classes in $H^3(X)$. Their method is geometric, using the Fredholm operator model for the spaces which define $K$-theory. Homotopically, this amounts to a multiplicative map from $K(Z, 2)$ to the space of units of $K$-theory, $GL_1(K)$. In joint work with Hisham Sati, we extended this construction to higher-chromatic versions of $K$-theory, Morava’s $E$-theories, $E_n$. We computed the space of $E$-infinity maps from $K(Z, n + 1)$ to $GL_1(E_n)$, thereby introducing a natural form of twisted $E$-theory. I will talk about these constructions and subsequent work which applies them to the study of the stable homotopy groups of the $(K(n)$-local) sphere.