Consider any stable $\infty$-category $\mathcal{C}$: Examples include $\text{DbCoh}(X)$, or the category of modules over some ring spectrum. We generalize the notion of a Bridgeland stability condition for a triangulated category to one for $\mathcal{C}$, and under some assumptions, the space of stability conditions is a complex manifold. For every stability condition $\sigma$, one can obtain a filtration on the algebraic $K$-theory of $\mathcal{C}$. These filtrations vary on the complex manifold only along real codimension 1 “walls” inside the complex manifold, and there should be a “wall-crossing formula” relating the $E_2$ pages of the spectral sequence associated to a filtration. I started looking into this because I wanted to encode Hall-algebra-like structures on the Ran space of the circle, so I will discuss that as motivation first.