Given a finite spectrum of type $n$, explicit $v_n$ self-maps are more easily constructed if that spectrum is a ring spectrum, by which I mean the spectrum is provided with a pairing which has a two-sided unit but is not necessarily homotopy commutative or homotopy associative. If in addition, the spectrum is homotopy associative and homotopy commutative, one can sometimes say more.

Twenty five years ago I proved that if $X$ is a finite ring spectrum of type $n$, then there exists a $v_n$ self-map $f$ such that the cofiber $X(f^i)$ of the self-map $f^i$ is a ring spectrum for any $i$, and the pairing on $X(f^i)$ extends the pairing on $X$. In this talk, I will discuss my recent result that if $X$ is higher homotopy commutative up to some finite order, then $f$ may be chosen so that this higher homotopy commutative structure may be extended to such a structure on $X(f^i)$. 