For $G$ a finite group with Sylow subgroup $S$, the conjugation action of $G$ on the subgroups of $S$ gives rise to the data of a saturated fusion system $\mathcal{F}_S(G)$ on $S$. On the other hand, $S$ acts on $G$ by left and right multiplication. The resulting $(S,S)$-biset $SGS$ turns out to contain much of the same information as $\mathcal{F}_S(G)$, in that the biset determines the fusion system, but not conversely.

These notions can be abstracted to make no reference to the ambient group $G$, resulting in an abstract saturated fusion system $\mathcal{F}$ on $S$ and a characteristic biset $\Omega$ for $\mathcal{F}$. Again, $\Omega$ determines $\mathcal{F}$, but each $\mathcal{F}$ has many associated characteristic bisets.

This talk will focus on the failure of a saturated fusion system to uniquely determine a characteristic biset. We will show that there is a parametrization of all characteristic bisets for a fixed $\mathcal{F}$, which will have as a consequence the surprising result that each saturated fusion system has a unique minimal associated characteristic biset.