We prove that every functor defined on dg categories which is derived Morita invariant, localizing, and $\mathbb{A}^1$-homotopy invariant, satisfies the fundamental theorem. As an application, we recover, in a unified and conceptual way, Weibel and Kassel’s fundamental theorems in homotopy algebraic K-theory, and periodic cyclic homology, respectively.