It has been observed that certain localizations of the spectrum of topological modular forms tmf are self-dual (Mahowald-Rezk, Gross-Hopkins). We provide an integral explanation of these results that is internal to the geometry of the (compactified) moduli stack of elliptic curves $\mathcal{M}_{\text{ell}}$ yet is only true in the derived setting. When $p$ is inverted, choice of level-$p$-structure for an elliptic curve provides a geometrically well-behaved cover of $\mathcal{M}_{\text{ell}}$, which allows one to consider tmf as the homotopy fixed points of $\text{tmf}(p)$, topological modular forms with level-$p$-structure, under a natural action by $\text{GL}_2(\mathbb{Z}/p)$. Specializing to $p = 2$ or $p = 3$ we obtain that as a result of Grothendieck-Serre duality, $\text{tmf}(p)$ is self dual. The vanishing of the associated Tate spectra then makes tmf itself Anderson self-dual.