Topology Seminar

Gabriel Katz

of MIT will be speaking on

Morse Theory, gradient flows, concavity, and complexity on manifolds with boundary

on August 2 at 4:30 in
MIT Room 2-131

Let $f : X \to \mathbb{R}$ be a Morse function on a manifold X and v its gradient-like vector field. Classically, the topology of a closed X can be described in terms of the spaces of v-trajectories that link the singular points of f. On manifolds with boundary, the situation is somewhat different: there, a massive set of nonsingular functions is available. For such Morse data (f, v), the interactions of the gradient flow with the boundary dX take central stage. We will introduce and measure the convexity and concavity of a v-flow relative to dX. "Some manifolds are intrinsically more concave than others with respect to any gradient flow" is the main slogan of the talk. Stated differently, the intrinsic concavity of X is a reflection of its complexity. We will explain how this approach leads to new topological invariants, both of the flow v and of the manifold X. In 3D, we have a good grasp of these invariants and their connection to the classification of 3-folds.