Topology Seminar

Gabriel Katz
of MIT will be speaking on

Morse Theory, gradient flows, concavity, and complexity on manifolds with boundary

on August 2 at 4:30 in
MIT Room 2-131

Let $f : X \to \mathbb{R}$ be a Morse function on a manifold $X$ and $\nu$ its gradient-like vector field. Classically, the topology of a closed $X$ can be described in terms of the spaces of $\nu$-trajectories that link the singular points of $f$. On manifolds with boundary, the situation is somewhat different: there, a massive set of nonsingular functions is available. For such Morse data $(f, \nu)$, the interactions of the gradient flow with the boundary $dX$ take central stage. We will introduce and measure the convexity and concavity of a $\nu$-flow relative to $dX$. “Some manifolds are intrinsically more concave than others with respect to any gradient flow” is the main slogan of the talk. Stated differently, the intrinsic concavity of $X$ is a reflection of its complexity. We will explain how this approach leads to new topological invariants, both of the flow $\nu$ and of the manifold $X$. In 3D, we have a good grasp of these invariants and their connection to the classification of 3-folds.