I will propose a simple and combinatorial $E_n$-operad which is built out of finite posets indexing a stratification of configuration spaces of points in an $n$-disk. This poset is constructed from a category $\theta_n$ which has recently become an important player for modeling weak $n$-categories (Joyal, Berger, Rezk). The techniques involved use the formalism of quasi-categories (Lurie). This project is joint with Richard Hepworth (Copenhagen) and is a work in progress.

One (nearly achieved) goal is to directly and geometrically relate three well-developed methods for recognizing $n$-fold loop spaces: as certain algebras over the little $n$-disk operad, as certain algebras over the Barratt-Eccles $E_n$ operad (via the Smith filtration), and as certain presheaves on $\theta_n$ (Berger). A version of Dunn’s additivity theorem becomes a formal consequence of the setup. A farther away goal is to imitate the construction of topological chiral homology using this proposed $E_n$-operad. This should have the benefit of making topological chiral homology (and possibly other field theories) more prepared for computations.