The symmetric groups $S_p$ are considered with the norm induced by the word length (with respect to transpositions as generators). This gives a filtration of their classifying spaces. Furthermore, using certain deletion functions $S_p \to S_{p-1}$ the family of all symmetric groups can be regarded as filtered simplicial object. We show: in its realization, the stratum for norm equal to $h$ has several components, each being homeomorphic to a vector bundle over the moduli space $M_{g, m}$ of genus $g$ surfaces with one boundary curve and $m$ punctures (for $h = 3D2g + m$).