Let $R$ be an associative ring spectrum. I shall describe several new constructions of the $R$-module Thom spectrum associated to a map $f : X \to \text{BGL}_1R$. The space $\text{BGL}_1R$ classifies the twists of $R$-theory, and to a fibration of manifolds $g : Y \to X$ I shall associated an Umkehr map $g_!$ from the $fg$-twisted $R$-theory of $Y$ to the $f$-twisted $R$-theory of $X$. In the case of K-theory, this twisted Umkehr map appears in the study of $D$-brane charge. I shall review this story, and then discuss the analogous construction for TMF.