We will describe how (multivariable) manifold calculus of functors can be used for studying classical knots and links. In particular, this theory yields a classification of finite type invariants and Milnor invariants of knots, links, homotopy links, and braids. Another novelty is that a certain cosimplicial variant of manifold calculus provides a way for studying knots and links in a homotopy-theoretic framework. Higher-dimensional analogs will also be discussed. This is joint work with Brian Munson.