18.03 Exercises

8: Extra Problems

8A. Bifurcation Diagrams

8A-1. Suppose that a population of variable size (in some suitable units) $P(t)$ follows the growth law $\dfrac{dP}{dt} = -P^3 + 12P^2 - 36P + r$, where r is a constant replenishment rate. Without solving the DE explicitly:

a) Let $r = 0$. Find all critical points and classify each according to its stability type using a phase-line diagram. Sketch some representative integral curves.

b) What is the smallest value of r such that the population always stabilizes at a size greater than 4, no matter what the size of the initial population?

c) Sketch the P vs. r bifurcation diagram.

8B. Frequency Response

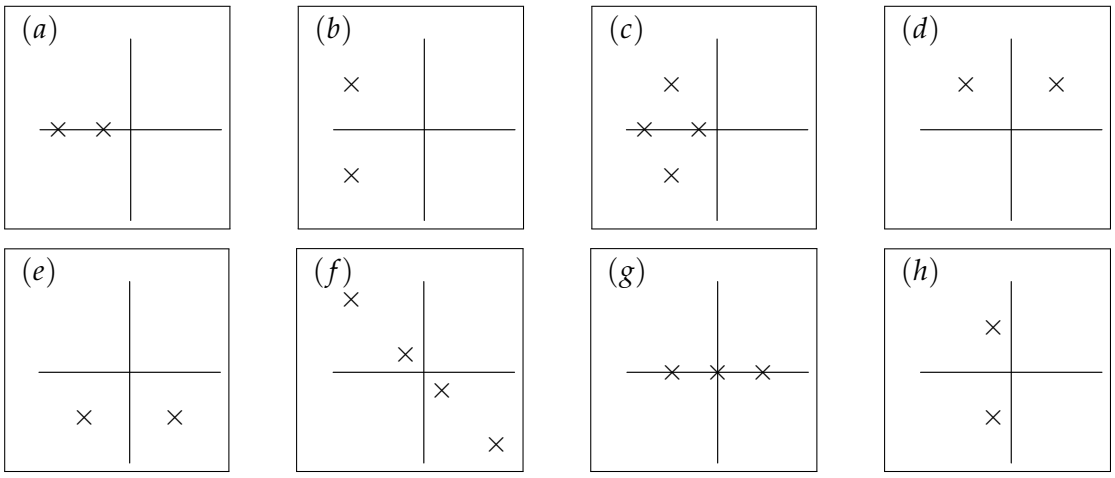
8B-1. For each of the following systems, use your calculus graphing skills to plot the graph of the amplitude response (i.e. gain vs. ω). If there is a resonant frequency say what it is.

a) $x'' + x' + 7x = F_0 \cos \omega t$.

b) $x'' + 8x' + 7x = F_0 \cos \omega t$.

8C. Pole Diagrams

8C-1. Consider the following pole diagrams for some linear time invariant systems $P(D)x = f(t)$.



a) Which of the systems are stable?

b) For which systems are *all* of the non-zero solutions to the homogeneous equation oscillatory?

c) For which systems are *none* of the non-zero solutions to the homogeneous equation oscillatory?

d) For which systems does $P(D)$ have real coefficients?

e) Comparing b and c, for which one does the weight function decay faster. (Assume both plots are on the same scale.)

f) Give the order of each of the systems.

g) Give a possible polynomial $P(D)$ that would have pole diagram (a). Do the same thing for (b) and (c).

h) Comparing (b) and (h) which has the largest possible response to input of the form $\cos \omega t$?

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18.03 Notes and Exercises