SOLUTIONS TO 18.01 REVIEW PROBLEMS

Unit 1: Differentiation

R1-0.

a) $\dfrac{-\gamma nRT}{V^{\gamma+1}}$ b) $\dfrac{m_0 v}{c^2(1-v^2/c^2)^{3/2}}$ c) $\dfrac{c\omega_0(2k+1)}{\beta^2}$

R1-1

a) $\dfrac{(x+1)\cos x - \sin x}{(x+1)^2}$ b) $\dfrac{\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}}$ c) $x^{1/3}\sec^2 x + \dfrac{1}{3}x^{-2/3}\tan x$

d) $\dfrac{\frac{3x^2}{2}+2x-1}{(\sqrt{x+1})^3}$ e) $\dfrac{-\sin\sqrt{x^2+1}\times x}{\sqrt{x^2+1}}$ f) $\dfrac{(3\cos^2\sqrt{x^2+1})(-\sin\sqrt{x^2+1})x}{\sqrt{x^2+1}}$

g) $3x^2\sec^2(x^3)$ h) $\sec^2(3x+1)+6x\sec^2(3x+1)\tan(3x+1)$

R1-2 $(0,-15)$

R1-3 $y = 4x - 10$

R1-4 Hint: Use addition formula to expand $\sin(x+\Delta x)$.

R1-5 a) $x = \pm 1$. b) $x = 0$

R1-6 $(1,-2)$ and $(-1,2)$

R1-7 See Simmons, sec. 3.2

R1-8 Hint: Differentiate implicitly the equation $y^5 = x$.

R1-10 a) $c + d = 2$ b) $c = 2, d = 0$

R1-11 a) cf. p. 75, Simmons. b) false; $f(x) = |x|$

R1-12 a) $b = -a\pi$ b) $a = -1$, $b = \pi$

R1-13 Let $4x = t$. $\displaystyle\lim_{x\to 0}\frac{\sin(4x)}{x} = \lim_{\frac{t}{4}\to 0}\frac{4\sin(t)}{t} = 4\lim_{t\to 0}\frac{\sin(t)}{t} = 4$

Unit 2: Applications of Differentiation

R2-2 a) $20\sqrt{5}$ ft/sec b) $50\sqrt{2}$ ft/sec

R2-3 5 **R2-4** a) 1 b) 1 c) $\dfrac{1}{10}$

R2-5 a) false b) cf. p. 801—802 (1), (2), (3) Simmons. **R2-6** 1

R2-7 $r = (2\pi)^{-1/3}$, $h = (4/\pi)^{1/3}$

R2-8 b) $\frac{6}{5} < f(2) < \frac{3}{2}$

R2-9 a) This is true, use mean value theorem. b) This is false ; try x^3.

R2-10 a) see graph b) $f(x)$ must be discontinuous c) $f(x)$ is discontinuous d) $|x|$

Unit 3: Integration

R3-1 2, $\frac{14}{3}, \frac{3}{2}$

R3-2 Yes. Hint: Find the time it takes him to reach the bottom of the hill, and find his speed at that instant.

R3-3 $\frac{8}{3}$ **R3-4** a) 16 b) $1 + \frac{3\sqrt{2}}{2}$ c) $(36)^{1/3}$

R3-5 $\frac{\pi}{4}$ **R3-6** $\leq 75,000,050$ **R3-7** $\frac{9}{2}$ **R3-8** 11.46

R3-10 7.566... **R3-12** 6 **R3-14** $4x - 3x^2$

Unit 4: Applications of Integration

R4-1. $\pi \times 197\frac{11}{15}$ **R4-2.** $\frac{128}{3}$ **R4-3.** 12

R4-5. 1944 ft.lbs **R4-6.** $2a^2$ **R4-7.** 8π

R4-8. 8. $\pi \times 9.63\dot{6}...$ **R4-9.** Another hour; never.

R4-10 b) hint: write $\int_1^{ab} f(t)dt = \int_1^a f(t)dt + \int_a^{ab} f(t)dt$

Unit 5: Integration Techniques

R5-1 $x^{1/x}(\frac{1-\ln x}{x^2}), e^{x^2}(\frac{2}{x} + 2x\ln(x^2)), \frac{1}{1+x^2}$

R5-2 a) $\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + c$ b) $\frac{e^x}{2}(\sin x - \cos x) + c$

R5-3 a) $\tan^{-1}(e^x) + c$ b) $\frac{1}{3}\ln\frac{(x-1)^2}{|x^2+x+1|} + c$ c) $2x^2 + 8x + 16\ln(x-2) + c$

R5-4 a) $\frac{1}{2}(\tan^{-1} x + \frac{x-1}{1+x^2}) + c$ b) $x^2\sin x + 2x\cos x - 2\sin x + c$

R5-5 $\frac{5\pi}{32}$