What is a fingerprint?
What is a mathematical fingerprint?
Fingerprint databases for theorems
Accumulation of evidence
Judgement time

Fingerprinting Richard Stanley

Bridget Eileen Tenner

DePaul University

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Example

a unique identifier

Properties of a fingerprint:

- small

- canonical

- language-independent

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What do we do with fingerprints?

1. Catalogue them in a <u>searchable</u> fingerprint database.

2. Query the database.

3. Solve crimes!

Who stole the cookies from the cookie jar?

What is a <u>mathematical</u> fingerprint?

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Example

$0, 1, 1, 2, 3, 5, 8, 13, \ldots$

a unique identifier

The nth term enumerates

- subsets of $[n-2]$ containing no consecutive integers
- domino tilings of a $2 \times (n-1)$ rectangle

Properties of a mathematical fingerprint:

- small

- canonical

- language-independent
  e.g., numerical

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What do we do with mathematical fingerprints?

1. Catalogue them in a <u>searchable</u> fingerprint database.

2. Query the database.

3. Solve mathematical problems!

What is the relationship between subsets of $[n-2]$ containing no consecutive integers, and domino tilings of a $2 \times (n-1)$ rectangle?

They are in bijection with each other.

Fingerprint databases for theorems

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Why bother?

Suppose mathematician M has just proved theorem T.

How can M determine if her result is truly new, or if T (or some equivalent reformulation of T) already exists in the literature?

Wouldn't it be great if M could encode T in some

small, canonical, language-independent

manner, and then search for that encoding in a database?!

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How does a database encode theorems?

In addition to the fingerprint, each entry includes fields like name, comments, formula, references, ...

Thus each entry in the database is a theorem, or a collection of theorems, associated to its particular fingerprint.

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A fingerprint database for theorems should be . . .

- language-independent

  numerical, canonical, no special vocab

- full of references

  literature, cross-reference among databases, code, links

- collaborative

  accessible, open

- optimistic

  false positives are okay, but no false negatives

  And the fingerprints should be small, canonical, and language-independent.

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What are some good mathematical fingerprints?

- integer sequences
  OEIS
- permutation patterns
  DPPA
- statistics for combinatorial objects
  FindStat
- finite simple groups
  ATLAS
- smooth, projective, irred curves of genus g over field of q elts
  manYPoints
- hypergeometric series in canonical form
  WZ method and Digital Library of Mathematical Functions
- graphs
  what should be the fingerprint??

Accumulation of evidence

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Integer sequences – OEIS

420 results found for query ref:stanley in OEIS as of June 16, 2014

A003431: number of connected irreducible posets with n labeled points [RPS]

A192721: number of pairs of permutations in $S_n \times S_n$ with k common descents [RPS]

In a different direction: 207 combinatorial interpretations of Catalan numbers as of June 16, 2014

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Permutation patterns – DPPA

Av(2143) = permutations w for which Red(w) and standard Young tableaux of shape $\lambda(w)$ are equinumerous [RPS]

Av(321) = fully commutative permutations [Billey-Jockusch-RPS]

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Not-yet-existing databases

A.k.a. homework for the audience!

- Combinatorial Hopf algebras
  via OEIS/FindStat hybrid?
- Chess problems
  via pieces? positions? restrictions?
- Catalan objects
  via symmetry?
- Tiling problems
  via tiles? symmetry? topology?
- Conjectures, open problems, attempts and ideas!

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The verdict

mathematics

GUILTY AS CHARGED