IN THE THEORY OF PARTITIONS, VARIOUS PARAMETERS (OR PARTITION STATISTICS) HAVE PLAYED AN IMPORTANT ROLE.

THIS BEGAN WITH EULER WHO KEPT TRACK OF THE NUMBER OF PARTS OF EACH PARTITION.

IN THE THEORY OF PARTITIONS, VARIOUS PARAMETERS (OR PARTITION STATISTICS) HAVE PLAYED AN IMPORTANT ROLE.

THIS BEGAN WITH EULER WHO KEPT TRACK OF THE NUMBER OF PARTS OF EACH PARTITION.

THUS IF $p(m,n)$ IS THE NUMBER OF PARTITIONS OF n INTO m PARTS, THEN

$$\sum_{n,m\geq 0} p(m,n) z^m q^n$$

$$= \sum_{n=0}^{\infty} \frac{z^n q^n}{(1-q)(1-q^2)\cdots(1-q^n)}$$

$$= \frac{1}{(1-zq)(1-zq^2)(1-zq^3)\cdots\cdots}$$

THE CONJUGATION MAP (TO BE DISCUSSED LATER) REVEALS THAT $p(m,n)$ IS ALSO THE NUMBER OF PARTITIONS OF n WITH LARGEST PART EQUAL TO m.

AND IF $p_d(m,n)$

IS THE NUMBER OF PARTITIONS OF n INTO m <u>DISTINCT</u> PARTS, THEN

$$\sum_{m,n\geq 0} p_d(m,n) z^m q^n$$

$$= \sum_{n=0}^{\infty} \frac{z^n q^{n(n+1)/2}}{(1-q)(1-q^2)\cdots(1-q^n)}$$

$$= (1+zq)(1+zq^2)(1+zq^3)\cdots$$

FOR EXAMPLE,

$p(4,8) = 5$

$5+1+1+1$, $4+2+1+1$, $3+3+1+1$

$3+2+2+1$, $2+2+2+2$

AND $p_d(3,12) = 7$

$9+2+1$, $8+3+1$, $7+4+1$

$7+3+2$, $6+5+1$, $6+4+2$

$5+4+3$

H. GUPTA STUDIED PARTITIONS WITH SMALLEST PART EQUAL TO m. THE GEN. F$\underline{N}$. IS

$$\sum_{m=1}^{\infty} \frac{z^m q^m}{(1-q^m)(1-q^{m+1})\cdots\cdots}$$

THIS WAS USED BY GUPTA IN THE 1930'S TO CONSTRUCT PARTITION TABLES.

FREEMAN DYSON
DEFINED THE RANK
OF A PARTITION AS
THE LARGEST
PART MINUS THE
NUMBER OF
PARTS. THE
GENERATING
FUNCTION IS NOW:

$$\sum_{n=0}^{\infty} \frac{q^{n^2}}{(zq;q)_n (z^{-1}q;q)_n}$$

WHERE

$$(A;q)_n = (1-A)(1-Aq)\cdots(1-Aq^{n-1}).$$

DYSON USED THE RANK TO MAKE CONJECTURES ABOUT A COMBINATORIAL INTERPRETATION OF RAMANUJAN'S THEOREM: $5 \mid p(5n+4)$.

NAMELY, IF WE CLASSIFY EACH PARTITION OF $5n+4$ ACCORDING TO ITS RANK mod 5, THEN THIS SPLITS THE PARTITIONS OF $5n+4$ INTO 5 EQUINUMEROUS CLASSES.

(Atkin & Swinnerton-Dyer)

$n=4$

	RANK	Mod 5
4	3	3
$3+1$	1	1
$2+2$	0	0
$2+1+1$	-1	4
$1+1+1+1$	-3	2

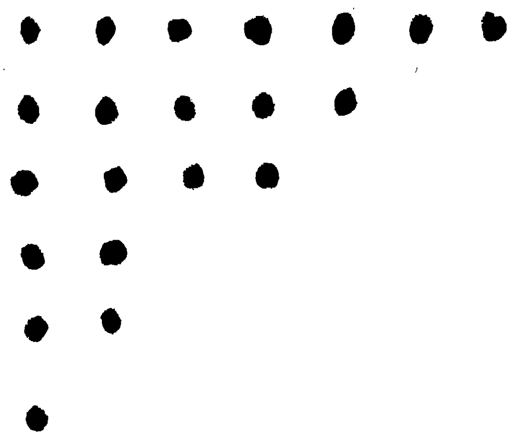
RAMANUJAN HAD THE RANK GENERATING FUNCTION IN THE "LOST NOTEBOOK," BUT HE NEVER MENTIONED THE RANK. FROM THIS STARTING POINT, AN AVALANCHE OF RESULTS ON MOCK THETA FUNCTIONS AND RELATED TOPICS HAS FOLLOWED.

RICHARD STANLEY HAS ALSO DEFINED A FRUITFUL PARTITION STATISTIC.

IN ORDER TO UNDERSTAND THE STANLEY STATISTIC, WE NEED THE CONCEPT OF THE CONJUGATE OF A PARTITION.

FERRERS GRAPH

$\pi: 7+5+4+2+2+1$



THE CONJUGATE COMES FROM THE COLUMNS

$\pi': 6+5+3+3+2+1+1$

LET $t(n)$ DENOTE THE NUMBER OF PARTITIONS, π, OF n FOR WHICH

$$\mathcal{O}(\pi) \equiv \mathcal{O}(\pi') \pmod{4}$$

WHERE $\mathcal{O}(\pi)$ IS THE NUMBER OF ODD PARTS OF π.

STANLEY'S THEOREM.

$$t(n) = \frac{1}{2}\left(p(n) + f(n)\right)$$

WHERE

$$\sum_{n=0}^{\infty} f(n) q^n$$

$$= \prod_{n \geq 1} \frac{(1+q^{2n-1})}{(1-q^{4n})(1+q^{4n-2})^2}.$$

COOL STUFF
FOLLOWS.

E.G.

$$5 \mid t(5n+4),$$

AND THERE ARE
IN DEPTH STUDIES
BY:

HOLLY SWISHER
BILL CHEN
$\vdots$

GIVEN HOW PRODUCTIVE THESE VARIOUS PARTITION STATISTICS HAVE BEEN, WE MAY ASK: "ARE THERE FURTHER PARTITION STATISTICS THAT HAVE BEEN COMPARATIVELY NEGLECTED?"

THE REMAINDER OF THIS TALK WILL FOCUS ON:

THE DIFFERENCE BETWEEN THE LARGEST PART AND THE SMALLEST PART.

(JOINT WORK WITH MATTHIAS BECK AND NEVILLE ROBBINS)

WE DEFINE

$p(n,t)$

$=$ # OF PARTITIONS OF n WHERE

$t =$ LARGEST PART $-$ SMALLEST PART

CLEARLY,

$p(n,0) = d(n)$.

A049820 n - d(n), where d(n) is the number of divisors of n (A000005). 43

0, 0, 1, 1, 3, 2, 5, 4, 6, 6, 9, 6, 11, 10, 11, 11, 15, 12, 17, 14, 17, 18, 21, 16, 22, 22, 23, 22, 27, 22, 29, 26, 29, 30, 31, 27, 35, 34, 35, 32, 39, 34, 41, 38, 39, 42, 45, 38, 46, 44, 47, 46, 51, 46, 51, 48, 53, 54, 57, 48, 59, 58, 57, 57, 61, 58, 65 (list; graph; refs; listen; history; text; internal format)

OFFSET 1,5

COMMENTS
a(n) = number of non-divisors of n in 1..n. [Jaroslav Krizek, Nov 14 2009]
Also equal to the number of partitions p of n such that max(p)-min(p) = 1. The number of partitions of n with max(p)-min(p) <= 1 is n; there is one with k parts for each 1 <= k <= n. max(p)-min(p) = 0 iff k divides n, leaving n-d(n) with a difference of 1. It is easiest to see this by looking at fixed k with increasing n: for k=3, starting with n=3 the partitions are [1,1,1], [2,1,1], [2,2,1], [2,2,2], [3,2,2], etc. - Giovanni Resta, Feb 06 2006 and Franklin T. Adams-Watters, Jan 30 2011.
a(n)=number of positive numbers in n-th row of array T given by A049816.
a(n) = SUM(A000007(A051731(n,k)): 1<=k<=n). [Reinhard Zumkeller, Mar 09 2010]
a(n) is the number of proper non-divisors of n. [Omar E. Pol, May 25 2010]
a(n) = A076627(n) / A000005(n). [Reinhard Zumkeller, Feb 06 2012]

LINKS
Reinhard Zumkeller, Table of n, a(n) for n = 1..10000

FORMULA
a(n) = sum(k=1, n, ceil(n/k)-floor(n/k)) - Benoit Cloitre, May 11 2003
G:=sum(x^(2*k+1)/(1-x^k)/(1-x^(k+1)),k=1..infinity); - Emeric Deutsch, Mar 01 2006
a(n) = A006590(n) - A006218(n) = A161886(n) - A000005(n) - A006218(n) + 1 for n >= 1. [Jaroslav Krizek, Nov 14 2009]
a(n+2) = sum of the n-th anti-diagonal of A225145. [Richard R. Forberg, May 02 2013]

EXAMPLE
a(7) = 5; the 5 non-divisors of 7 in 1..7 are 2, 3, 4, 5, and 6.
The 5 partitions of 7 with max(p) - min(p) = 1 are [4,3],[3,2,2],[2,2,2,1], [2,2,1,1,1] and [2,1,1,1,1,1]. - Emeric Deutsch, Mar 01 2006

MAPLE
```
with(numtheory); A049820 := n->n-sigma[0](n);
```

PROG
```
(PARI) a(n)=n-numdiv(n)
(Haskell)
a049820 n = n - a000005 n  -- Reinhard Zumkeller, Feb 06 2012
```

CROSSREFS
Cf. A000005, A062249.
Cf. A173540, A173541.

Sequence in context: A062327 A075491 A089279 * A109712 A095049 A118209
Adjacent sequences: A049817 A049818 A049819 * A049821 A049822 A049823

KEYWORD nonn,easy

$$p(n,1) = n - d(n)$$

BECAUSE THE PARTITIONS COUNTED BY

$$p(n,0) + p(n,1)$$

CONTAIN EXACTLY ONE SAMPLE WITH k PARTS $1 \leq k \leq n$.

EX. $n=9$

$9,\ 5+4,\ 3+3+3,\ 3+2+2+2,$

$2+2+2+2+1,\ 2+2+2+1+1+1,$

$2+2+1+1+1+1+1,$

$2+1+1+1+1+1+1+1,$

$1+1+1+1+1+1+1+1+1$

NOW DELETE

$9,\ 3+3+3,$

$1+1+1+1+1+1+1+1+1$

AND

$p(9,1) = 9-3 = 6.$

A117142 Number of partitions of n in which any two parts differ by at most 2. 8

1, 2, 3, 5, 6, 9, 10, 14, 15, 20, 21, 27, 28, 35, 36, 44, 45, 54, 55, 65, 66, 77, 78, 90, 91, 104, 105, 119, 120, 135, 136, 152, 153, 170, 171, 189, 190, 209, 210, 230, 231, 252, 253, 275, 276, 299, 300, 324, 325, 350, 351, 377, 378, 405, 406, 434, 435, 464, 465 (list; graph; refs; listen; history; text; internal format)

OFFSET 1,2

COMMENTS Equals row sums of triangle A177991. -Gary W. Adamson, May 16 2010
Positive numbers that are either triangular (A000217) or triangular minus 1 (A000096). - Jon E. Schoenfield, Jun 12 2010

LINKS Alois P. Heinz, Table of n, a(n) for n = 1..1000

FORMULA G.g.: sum(k>=1, x^k/((1-x^k)*(1-x^(k+1))*(1-x^(k+2)))). More generally, the g.f. of the number of partitions in which any two parts differ by at most b is sum(k>=1, x^k/prod(j=k..k+b, 1-x^j)).
a(n) = (2*n^2 + 10*n + 3 + (-1)^n * (2*n - 3))/16. - Birkas Gyorgy, Feb 20 2011
G.f.: (1+x)/(1-x)/(Q(0)-x^2-x^3), where Q(k) = 1 + x^2 + x^3 + k*x*(1+x^2) - x^2*(1 + x*(k+2))*(1+k*x)/Q(k+1) ; (continued fraction). - Sergei N. Gladkovskii, Jan 05 2014

EXAMPLE a(6) = 9 because we have
1: [6],
2: [4,2],
3: [3,3],
4: [3,2,1],
5: [3,1,1,1],
6: [2,2,2],
7: [2,2,1,1],
8: [2,1,1,1,1], and
9: [1,1,1,1,1,1]
([5,1] and [4,1,1] do not qualify).

MAPLE
```
g:=sum(x^k/(1-x^k)/(1-x^(k+1))/(1-x^(k+2)), k=1..75): gser:=series(g, x=0, 70): seq(coeff(gser, x^n), n=1..65); with(combinat): for n from 1 to 7 do P:=partition(n): A:={}: for j from 1 to nops(P) do if P[j][nops(P[j])]-P[j][1]<3 then A:=A union {P[j]} else A:=A fi od: print(A); od: # this program yields the partitions
```

MATHEMATICA
```
Table[Count[IntegerPartitions[n], _?(Max[#] - Min[#] <= 2 &)], {n, 30}] (* Birkas Gyorgy, Feb 20 2011 *)
Table[(2 n^2 + 10 n + 3 + (-1)^n (2 n - 3))/16, {n, 30}] (* Birkas Gyorgy, Feb 20 2011 *)
```

CROSSREFS Cf. A117143.
Cf. A177991 - Gary W. Adamson, May 16 2010

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A008805 Search Hints

(Greetings from Sequences!)

Search: **a008805**

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A008805 Triangular numbers repeated.

1, 1, 3, 3, 6, 6, 10, 10, 15, 15, 21, 21, 28, 28, 36, 36, 45, 45, 55, 55, 66, 66, 7 78, 91, 91, 105, 105, 120, 120, 136, 136, 153, 153, 171, 171, 190, 190, 210, 210, 2 231, 253, 253, 276, 276, 300, 300, 325 (list;)

OFFSET 0,3

COMMENTS

Number of choices for nonnegative integers x,y,z such that x and y are and x+y+z = n.

a(n) = number of partitions of n+4 such that the differences between greatest and smallest parts are 2: a(n-4) = A097364(n,2) for n>3. - Reinhard Zumkeller, Aug 09 2004

a(n) = A (n-2,n)*(-1)^floor((n+1)/2) for n>1. - Jun 01 2005

For n>=i,i=4,5, a(n-i) is the number of incongruent two-color bracelet n beads, i from them are black (Cf. ,), having a diam of symmetry. - elev, May 03 2011

Prefixing A008805 by 0,0,0,0 gives the sequence c(0), c(1),... definec c(n)=number of (w,x,y) such that w = 2x+2y, where w,x,y are all in {1,...,n}; see . [Clark Kimberling, Apr 15 2012]

Partial sums of positive terms of A . - , Jul C 2012

The sum of the first parts of the non-decreasing partitions of n+2 int exactly two parts, n >= 0. - , Jun 08 2013

Number of the distinct symmetric pentagons in a regular n-gon, see illustration for some small n in links. - Ngaokrajang, Jun 25

a(n) is the number of nonnegative integer solutions to the equation x z = n such that x + y <= z. For example a(4) = 6 because we have: C = 0+1+3 = 0+2+2 = 1+0+3 = 1+1+2 = 2+0+2. - , Jul 09

a(n) = number of distinct opening moves in n X n tic-tac-toe. - y, Sep 04 2013

REFERENCES

H. D. Brunk, An Introduction to Mathematical Statistics, Ginn, Boston, 1960; p. 360.

LINKS

Vincenzo Librandi,

P. Flajolet and R. Sedgewick, , 2009; see page 4

Kival Ngaokrajang, 6..1

V. Shevelev,

NOTE
REINHARD ZUMKELLER

$$p(n,2) = \binom{\lfloor \frac{n}{2} \rfloor}{2}$$

OR

$$p(n,2) = \begin{cases} \binom{j}{2} & n = 2j \\ \binom{j}{2} & n = 2j+1 \end{cases}$$

$p(n,2)$ IS A
PSEUDO-POLYNOMIAL

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1,1,3,3,7,7,12 Search Hints

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Displaying 1-1 of 1 result found. page 1

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A128508 Number of partitions p of n such that max(p)-min(p)=3. +20 3

0, 0, 0, 0, 0, **1**, **1**, **3**, **3**, **7**, **7**, **12**, 14, 20, 22, 32, 34, 45, 51, 63, 69, 87, 93, 112, 124, 144, 156, 184, 196, 225, 245, 275, 295, 335, 355, 396, 426, 468, 498, 552, 582, 637, 679, 735, 777, 847, 889, 960, 1016, 1088, 1144, 1232, 1288, 1377, 1449, 1539, 1611, 1719

(list; graph; refs; listen; history; text; internal format)

OFFSET	0,8
COMMENTS	See A008805 and A049820 for the numbers of partitions p of n such that max(p)-min(p)=1 or 2, respectively.
LINKS	Alois P. Heinz, Table of n, a(n) for n = 0..1000
FORMULA	Conjecture. a(1)=0 and, for n>1, a(n+1)=a(n)+d(n), where d(n) is defined as follows: d=0,0,0,1,0 for n=1,...,5 and, for n>5, d(n)=d(n-2)+1 if n=6k or n=6k+4, d(n)=d(n-2) if n=6k+1 or n=6k+3, d(n)=d(n-2)+2Floor[n/6] if n=6k+2 and d(n)=d(n-5) if n=6k+5. G.f. for number of partitions p of n such that max(p)-min(p) = m is Sum_{k>0} x^(2*k+m)/Product_{i=0..m} (1-x^(k+i)). - Vladeta Jovovic, Jul 04 2007 a(n) = A097364(n,3) = A116685(n,3) = A117143(n) - A117142(n). - Alois P. Heinz, Nov 02 2012
MATHEMATICA	np[n_]:=Length[Select[IntegerPartitions[n], Max[#]-Min[#]==3&]]; Array[np, 60] (* Harvey P. Dale, Jul 02 2012 *)
CROSSREFS	Cf. A008805, A049820, A097364, A116685, A117142, A117143.
KEYWORD	nonn
AUTHOR	John W. Layman, May 07 2007
EXTENSIONS	More terms from Vladeta Jovovic, Jul 04 2007
STATUS	approved

page 1

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CONJECTURE

$$p(n,3) = \frac{1}{108} \times \begin{cases} n^3 - 18n, & n \equiv 0 \pmod{6} \\ n^3 - 3n + 2, & n \equiv 1 \pmod{6} \\ n^3 - 30n + 52, & n \equiv 2 \pmod{6} \\ n^3 + 9n - 54, & n \equiv 3 \pmod{6} \\ n^3 - 30n + 56, & n \equiv 4 \pmod{6} \\ n^3 - 3n - 2, & n \equiv 5 \pmod{6} \end{cases}$$

ANOTHER PSEUDO-POLYNOMIAL

IN SUMMARY,

$p(n,0)$ AND $p(n,1)$

<u>ARE</u> <u>NOT</u>

PSEUDO-POLYNOMIALS.

$p(n,1)$ AND $p(n,2)$

<u>ARE</u>

PSEUDO-POLYNOMIALS

GENERATING F$^{\underline{N}}$S:

$$\mathcal{P}_t(q) := \sum_{n\geq 1} p(n,t)\, q^n.$$

$$\mathcal{P}_0(q) = \sum_{n=1}^{\infty} \frac{q^n}{1-q^n}$$

$$\mathcal{P}_1(q) = \frac{q}{(1-q)^2} - \sum_{n=1}^{\infty} \frac{q^n}{1-q^n}$$

$$\mathcal{P}_2(q) = \frac{q^4}{(1-q)^3(1+q)^2}$$

$$\mathcal{P}_3(q) = \frac{q^5+q^6+q^7-q^8}{(1-q)^4(1+q)^2(1+q+q^2)^2}$$

IT TURNS OUT

$$\mathcal{P}_4(q) = \frac{q^6 + q^7 + 2q^8 - q^{11} - q^{12} + q^{13}}{(1-q)^5(1+q)^3(1+q^2)^2(1+q+q^2)^2}$$

"CURIOUSER AND CURIOUSER!"

THEOREM

(JOINT WITH BECK AND ROBBINS)

$$\mathcal{P}_t(q) = \frac{q^{t-1}(1-q)}{(1-q^t)(1-q^{t-1})}$$

$$- \frac{q^{t-1}}{(1-q^t)(1-q^{t-1})\ (q)_t}$$

$$+ \frac{q^t}{(1-q^{t-1})\ (q)_t},$$

WHERE

$$(q)_t = (1-q)(1-q^2)\cdots(1-q^t)$$

IDEA OF PROOF:

$$\mathcal{P}_t(q) =$$

$$\sum_{m\geq 1} \frac{q^m}{(1-q^m)} \frac{1}{(1-q^{m+1})} \cdots \frac{q^{m+t}}{(1-q^{m+t})}$$

$$= \frac{q^{t+2}}{(q)_{t+1}} \sum_{m\geq 0} \frac{(q)_m \, q^{2m}}{(q^{t+2})_m}$$

WHERE

$$(A)_m = (1-A)(1-Aq)\cdots(1-Aq^{m-1})$$

THE LATTER SERIES IS AN INSTANCE OF

$$F(a,b;t:q) = 1 + \sum_{m \geq 1} \frac{(aq;q)_m t^m}{(bq;q)_m}$$

STUDIED BY N.J. FINE IN: BASIC HYPERGEOMETRIC SERIES AND APPLICATIONS.

FINE PROVES (p. 5, eq. (6.3)):

$$F(a,b;t:q)$$

$$= \frac{(1-b)}{(1-t)} F\left(\frac{at}{b}, t; b : q\right)$$

(ACTUALLY AN INSTANCE OF HEINE'S 2nd TRANSFORMATION)

CONSEQUENTLY,

$$\mathcal{P}_t(q) = \frac{q^{t-1}(1-q)}{(1-q^t)(1-q^{t-1})(q)_t} \times \sum_{j=0}^{t-2} \begin{bmatrix} t \\ j+2 \end{bmatrix} (-1)^j q^{\binom{j+3}{2}}$$

WHERE

$$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{(q)_A}{(q)_B \, (q)_{A-B}},$$

the q-binomial coefficient.

AND

$$\sum_{j=0}^{t-2} \begin{bmatrix} t \\ j+2 \end{bmatrix} (-1)^j q^{\binom{j+3}{2}}$$

$$= (q)_t - 1 + q \begin{bmatrix} t \\ 1 \end{bmatrix},$$

AND THE THEOREM FOLLOWS.

ONCE ONE REALIZES THAT FINE'S THEOREM IS THE ESSENTIAL ELEMENT, IT IS NATURAL TO EXTEND OUR THEOREM TO <u>PARTITIONS</u> <u>WITH</u> <u>SPECIFIED</u> <u>DISTANCES</u>.

LET $p(n, t_1, t_2, \ldots, t_k)$ BE THE # OF PTNS OF n SUCH THAT IF σ IS THE SMALLEST PART, THEN

$\sigma + t_1 + t_2 + \cdots + t_n$

IS THE LARGEST AND EACH OF

$\sigma + t_1,\ \sigma + t_1 + t_2, \ldots,$

$\sigma + t_1 + t_2 + \cdots + t_{n-1}$ IS ALSO A PART.

DEFINE

$$\wp_{t_1,\dots,t_k}(q) := \sum_{n\geq 1} p(n,t_1,\dots,t_k)\, q^n.$$

THEOREM

(JOINT WITH BECK & ROBBINS)

$$\wp_{t_1,\dots,t_k}(q) = \frac{(-1)^k q^{T-\binom{k+1}{2}}\left(\sum_{j=0}^{k} \begin{bmatrix} t \\ j \end{bmatrix} (-1)^j q^{\binom{j+1}{2}} - (q)_t\right)}{\begin{bmatrix} t-1 \\ k \end{bmatrix}(1-q^t)\,(q)_t}$$

where

$$t = t_1 + t_2 + \cdots + t_k$$

$$T = k t_1 + (k-1) t_2 + \cdots + 2 t_{k-1} + t_k$$