Physical Mathematics Seminar

Critical Gels and the Fractional Calculus of Foods and Soft Squishy Materials

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ABSTRACT:

Many soft materials including foods, consumer products, biopolymer gels & associative polymer networks are characterized by multi-scale microstructures and exhibit power-law responses in canonical rheological experiments such as Small Amplitude Oscillatory Shear (SAOS) and creep. Even in the linear limit of small deformations it is difficult to describe the material response of such systems quantitatively within the classical viscoelastic framework of springs and dashpots – which give rise universally to Maxwell-Debye exponential responses. Instead empirical measures of quantities such as 'firmness', 'tackiness' etc. are often used to describe and compare material responses. Scott Blair, who was present with Bingham at the founding of the Society of Rheology, first argued that such measures are best thought of as *'quasi-properties'* that capture a snapshot of the underlying dynamical processes in these complex materials. We show that the language of fractional calculus and the concept of the 'spring-pot' element together provide a useful framework that is especially well-suited for modeling and quantifying the rheological response of complex fluids and power-law materials. We first illustrate the general utility of this approach by reviewing fractional differential forms of the well-known Maxwell and Kelvin-Voigt models and use them to quantify small-amplitude oscillatory shear responses and creep response in a range of soft materials including gluten gels, skin and soft tissue, filled polymer melts, hydrogen-bonded biopolymer networks and the complex interfacial rheological properties of acacia gum and serum albumins [1]. The fractional exponents that characterize the dynamic material response can also be connected directly with the scaling exponents from microstructural models such as the Rouse model and the Soft Glassy Rheology (SGR) model. Having determined the quasi-properties that quantify the *linear viscoelastic* material response of a power-law gel in a concise form, we show that a fractional K-BKZ framework combining a Mittag-Leffler relaxation kernel with a strain-damping function can be used to quantitatively describe the *nonlinear* viscometric properties of such materials [2]. The material parameters extracted from this framework also prove especially useful for ranking, inter-comparing and formulating complex microstructured fluid materials such as liquid foodstuffs used in treating oral dysphagia (swallowing disorders) [3, 4].

[1] Jaishankar, A. and G. H. McKinley, Power-Law Rheology at the Bulk and the Interface: Quasi-properties and Fractional Constitutive Equations. *Proc. Roy. Soc. A*, (2013): **469**(2149), Online Oct. 24; DOI: 10.1098/rspa.2012.0284

[2] Jaishankar, A. and G. H. McKinley, A Fractional K-BKZ Constitutive Formulation for Describing the Nonlinear Rheology of Multiscale Complex Fluids, *J. Rheology*, (2014), **58**(6), 1751-1788.

[3] Faber, T. J, Jaishankar, A., McKinley, G.H., Describing the Firmness, Springiness and Rubberiness of Food Gels using Fractional Calculus. Part I; Theoretical Framework, *Food Hydrocolloids*, (2017), **62**, 311-324.

[4] C. E. Wagner, Barbati, A., Engmann, J., Burbidge A.S., McKinley G.H., Quantifying the consistency and rheology of liquid foods using fractional calculus, *Food Hydrocolloids* (2017), **69**, 242-254.

TUESDAY, MAY 15, 2018
2:30 PM – 3:30 PM
Building 2, Room 136

Reception following in Building 2, Room 290
(Math Dept. Common Room)

http://math.mit.edu/seminars/pms/

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