Physical Mathematics Seminar

A Dynamic Approach to Spatially Localized Planar Patterns

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ABSTRACT:

Spatially localized structures, in which a spatially oscillatory pattern on a finite spatial range connects to a trivial homogeneous solution outside this range, have been observed in numerous physical contexts, including cellular buckling, plane Couette flow, vegetation patterns, optical cavity solitons, crime hotspots, and many others. Despite the widely disparate contexts in which they arise, the bifurcation diagrams of such patterns often exhibit similar snaking behavior, in which branches of symmetric solutions, connected by bifurcating branches of asymmetric solutions, wind back and forth between two limits of an appropriate parameter. Here we address the existence and stability of stationary localized solutions of parabolic PDEs, and the behavior of such solutions upon perturbation of the underlying system. We use the Swift–Hohenberg system for numerical illustration of our results.

We give a new proof of the existence of asymmetric localized structures, utilizing information about the underlying front structure and providing a unified approach to the existence of all localized structures. This enables a rigorous proof of the stability properties of both symmetric and asymmetric structures. We show that the temporal eigenvalues of localized structures in the right half plane are exponentially close to those of the front and back added with multiplicity, and importantly that the eigenvalue at the origin remains simple. We further address numerical results showing unexpected behavior of eigenvalues within the essential (or absolute) spectrum, and propose an analytical explanation of these results. We conclude by making both qualitative and quantitative predictions on the results of perturbative terms breaking symmetries and/or variational structure in PDE systems supporting localized snaking solutions. We catalogue possible topological changes to the associated bifurcation diagrams, and show that we can predict *a priori* both the perturbed bifurcation diagram and the drift speeds of particular solutions using only the original bifurcation diagram.

TUESDAY, MARCH 8, 2016
2:30 PM – 3:30 PM
Building 2, Room 136

Reception following in Building 2, Room 290
(Math Dept. Common Room)

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