Cells live in complex and dynamic environments. Adapting to changing environments often requires cells to perform complex information processing, and cells have developed elaborate signaling networks to accomplish this feat. These biochemical networks are ubiquitous in biology. They range from naturally occurring biochemical networks in bacteria and higher organisms, to sophisticated synthetic cellular circuits that rewire cells to perform complex computations in response to specific inputs. The tremendous advances in our ability to understand and manipulate cellular information processing networks raise fundamental questions about the physics of information processing in living systems. I will discuss recent work in this direction trying to understand the fundamental constraints placed by (nonequilibrium) thermodynamics on the ability of cellular circuits to process information and perform computations and discuss the implications of our results for the emerging field of synthetic biology.