PHYSICAL MATHEMATICS SEMINAR

SPECTRAL OPTIMIZATION PROBLEMS FOR ENGINEERING LONG-LIFETIME STATES

BRAXTON OSTING
Columbia University

ABSTRACT:

In many optical and quantum systems it is desirable to spatially confine energy in a particular mode for a long period of time. We discuss energy confinement in two energy-conserving extended systems.

(1) Consider a system governed by the wave equation with wave speed c(x), where c(x) is inhomogeneous within a bounded region and constant outside. We also assume that c(x) is bounded point-wise from above and below everywhere. The solution of the time-dependent wave equation with spatially-localized initial data spreads and decays with advancing time; the spatially-localized energy decays with time. The rate of decay can be measured in terms of the eigenvalues of the scattering resonance problem, a non-selfadjoint eigenvalue problem consisting of the time-harmonic wave (Helmholtz) equation with outgoing radiation condition at infinity. Specifically, the rate of localized energy decay is governed by the complex scattering eigenfrequency which is closest to the real axis. We study the structural design problem: Find c*(x) within an admissible class which has scattering frequency with minimal imaginary part. We formulate this problem as a constrained optimization problem and prove that an admissible optimal solution exists. We also show that the optimal structure is 'bang-bang' in the sense that if point-wise bounds are imposed on c(x), then the optimal wave-speed, c*(x), will achieve these bounds almost everywhere. In one-dimension, we establish a connection between c*(x) and the well-known class of Bragg structures, where c(x) is constant on intervals whose length is one-quarter of the effective wavelength.

(2) Consider a system governed by the time-dependent Schroedinger equation in its ground state. When subject to weak parametric forcing by an "ionizing field" (time-varying), the state decays with advancing time due to coupling of the bound state to radiation modes. The decay-rate of this metastable state is governed by Fermi's Golden Rule, which depends on the potential and the details of the forcing. We study the structural design problem: find the potential within an admissible class which minimizes Fermi's Golden Rule (maximizes the lifetime of the state). We formulate this problem as a constrained optimization problem and prove that an admissible optimal solution exists. Using computational methods, we investigate properties of locally optimal solutions.

This is joint work with Michael I. Weinstein.

TUESDAY, MAY 17, 2011
2:30 PM
Building 2, Room 105

Refreshments at 3:30 PM in Building 2, Room 290

Massachusetts Institute of Technology