ABSTRACT:

The RBF-FD (Radial Basis Function-Finite Difference method) can be considered as an efficient alternative for solving PDEs. It works very similarly to the finite difference (FD) method: differential operators at a given node are approximated as a weighted sum of the values of the sought function at some surrounding nodes. However, while in the FD method the unknown weights are computed using polynomial interpolation, in the local RBF method they are computed by fitting an RBF interpolant through a stencil of neighboring nodes. Most of the RBFs used in practical applications depend on a shape parameter, and there is much experimental evidence showing that the accuracy of the solution to a PDE strongly depends on the value of this parameter. We will show that there is a range of values of the shape parameter for which the RBF-FD formulas are significantly more accurate than the FD ones. Then, we will describe a novel technique to compute efficiently such value. The method is based on analytical approximations to the local truncation error. Two different strategies will be analyzed. The first one computes a constant value of the shape parameter which minimizes the norm of the error in the whole domain. The second computes a set of variable shape parameters which minimize the local truncation error in every node. Both strategies result in several orders of magnitude increase in accuracy with respect to standard FD.