ABSTRACT:

The geometric approach to mechanics serves as the theoretical underpinning of innovative control methodologies in geometric control theory. These techniques allow the attitude of satellites to be controlled using changes in its shape, as opposed to chemical propulsion, and are the basis for understanding the ability of a falling cat to always land on its feet, even when released in an inverted orientation.

We will discuss the application of geometric structure-preserving numerical schemes to the optimal control of mechanical systems. In particular, we consider Lie group variational integrators, which are based on a discretization of Hamilton's principle that preserves the Lie group structure of the configuration space. In contrast to traditional Lie group integrators, issues of equivariance and order-of-accuracy are independent of the choice of retraction in the variational formulation. The importance of simultaneously preserving the symplectic and Lie group properties is also demonstrated.

Recent extensions to homogeneous spaces yield intrinsic methods for Hamiltonian flows on the sphere, and have potential applications to the simulation of geometrically exact rods, structures and mechanisms. Extensions to Hamiltonian PDEs and uncertainty propagation on Lie groups using noncommutative harmonic analysis techniques will also be discussed.

We will place recent work in the context of progress towards a coherent theory of computational geometric mechanics and computational geometric control theory, which is concerned with developing a self-consistent discrete theory of differential geometry, mechanics, and control.

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