PHYSICAL MATHEMATICS SEMINAR

Modeling Heated Falling Films: From Low to Moderate Reynolds Numbers

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ABSTRACT:

Thin films flowing down inclines have a rich dynamics extensively studied for a long time since Kapitza's experimental and theoretical pioneering works at the end of the forties. Besides their importance for engineering applications (e.g. evaporators or chemical reactors), their interest mainly stems from the fact that their evolution is amenable to thorough theoretical analysis. This situation happens mostly due to the two-dimensional, long-wavelengthed, supercritical character of the primary instability mode that leads to hydrodynamic waves. Thickness modulations which develop over initially uniform films are usually spanwise homogeneous and slowly varying both in time and in space, which allows gradient expansions on which relies the lubrication approximation. The classical long-wave equation for the evolution of the film thickness, often referred to as the Benney equation, is an extensively used model that satisfactory describes the falling film dynamics as long as the Reynolds number remains small. A practical example including Marangoni effects and showing a rivulet instability will be discussed in this talk. However, the Benney equation suffers from singularity as the Reynolds number increases. This can be cured by combining a gradient expansion with a weighted integral method similar to that followed in the study of boundary layers. Hence deep analogies can be found in the transition to turbulence of each system, especially at the level of secondary instabilities.

The application of a systematic strategy to the problem of film flows is then shown here to lead to systems of equations of reduced dimensionality that capture the physical mechanisms quite faithfully, helping us to enlighten the observed dynamics by isolating the important physical effects. Additionally, a regularization procedure applied in the approach pushes away the validity range of a consistent three-field modeling of film flows in parameter space (i.e. valid for larger Reynolds numbers) by reducing the degrees of the dangerous nonlinear inertia terms. Having at one's disposal reliable low-dimension models, one was able to undergo a systematic analytical and numerical analysis. Consequently, complex wave patterns such that herringbone patterns, synchronously deformed fronts, oblique and V-shape solitary waves observed in various experimental data were reliably recovered. The talk will present the last advances in this field, including the extension to the heating case and thus including the Marangoni effect. The heating yields surface tension gradients that induce thermocapillary stresses on the free surface, thus affecting the stability and the evolution of the film. A phase diagram will be presented in this case, showing in particular a regime of spontaneous channeling of solitary pulses.

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2:30 PM
Building 2, Room 105

Refreshments at 3:30 PM in Building 2, Room 349
(Applied Math Common Room)

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