ON A CENTRAL LIMIT TYPE CONJECTURE FOR THE NODAL
STATISTICS OF QUANTUM GRAPHS

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Understanding statistical properties of Laplacian eigenfunctions in general and their
nodal sets in particular have an important role in the field of spectral geometry, and interest
both mathematicians and physicists. A quantum graph is a system of a metric graph with
a self-adjoint Schrödinger operator acting on it. In the case of quantum graphs it was
proven that the number of points on which each eigenfunction vanishes, also known as the
nodal count, is bounded away from the spectral position of the eigenvalue by a topological
quantity, the first Betti number of the graph. A remarkable result by Berkolaiko and
Weyand (with another proof for discrete graphs by Colin de Verdiere) showed that the
nodal surplus is equal to a magnetic stability index of the corresponding eigenvalue.

Both from the nodal count point of view and from the physical magnetic point of view,
it is interesting to consider the distribution of these indices over the spectrum. In our work
we show that such a density exists and define a nodal surplus distribution. Moreover this
distribution is symmetric, which allows to deduce the Betti number of a graph from its
nodal count. A further result proves that the distribution is binomial with parameter 1/2
for a certain large family of graphs. The binomial distribution satisfies the Central limit
theorem (CLT) and converge under appropriate normalization to normal distribution. A
numerical study indicates that the CLT convergence is independent of the specific choice
of the growing family of graphs. In my talk I will talk about our latest results extending
the number of families of graphs for which we can prove the CLT convergence.

This talk is based on joint works with Ram Band (Technion) and Gregory Berkoliako
(Texas A&M).