In 2000, Caffarelli showed that given a Gaussian distribution $\gamma$ on $\mathbb{R}^n$ and a log-concave perturbation of that Gaussian distribution $e^{-U}\gamma$ with $U$ convex, the optimal transport (for quadratic cost) that takes $\gamma$ to $e^{-U}\gamma$ is a 1-Lipschitz change of variable. Motivated by his result, we exploit the relationship between optimal transportation and the Monge-Ampère equation to investigate conditions under which one can find Lipschitz changes of variables between log-concave measures, the natural generalizations of Gaussian measures, and perturbations of these measures. This is joint work with Alessio Figalli and Yash Jhaveri.