A PROOF OF ONSAGERS CONJECTURE FOR THE
INCOMPRESSIBLE EULER EQUATIONS

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In an effort to explain how anomalous dissipation of energy occurs in hydro-
dynamic turbulence, Onsager conjectured in 1949 that weak solutions to the in-
compressible Euler equations may violate the law of conservation of energy if their
spatial regularity is below $1/3$-Hölder. I will discuss a proof of this conjecture that
shows that there are nonzero, $(1/3 - \epsilon)$-Hölder Euler flows in 3D that have compact
support in time. The construction is based on a method known as "convex integra-
tion," which has its origins in the work of Nash on isometric embeddings with low
codimension and low regularity. A version of this method was first developed for
the incompressible Euler equations by De Lellis and Székelyhidi to build Hölder-
continuous Euler flows that fail to conserve energy, and was later improved by Isett
and by Buckmaster-De Lellis-Székelyhidi to obtain further partial results towards
Onsager’s conjecture. The proof to be discussed of the full conjecture combines
a new ingredient in the convex integration scheme due to Daneri-Székelyhidi with
a new "gluing approximation" technique. The latter technique exploits a special
structure in the linearization of the incompressible Euler equations.