Title: The Sample Complexity of Multi-Reference Alignment

Abstract: How should one estimate a signal, given only access to noisy versions of the signal corrupted by unknown cyclic shifts? This simple problem has surprisingly broad applications, in fields from aircraft radar imaging to structural biology with the ultimate goal of understanding the sample complexity of Cryo-EM. We describe how this model can be viewed as a multivariate Gaussian mixture model whose centers belong to an orbit of a group of orthogonal transformations. This enables us to derive matching lower and upper bounds for the optimal rate of statistical estimation for the underlying signal. These bounds show a striking dependence on the signal-to-noise ratio of the problem. We also show how a tensor based method of moments can solve the problem efficiently. Based on joint work with Afonso Bandeira (NYU), Amelia Perry (MIT), Amit Singer (Princeton) and Jonathan Weed (MIT).