We are concerned with the questions of global regularity vs. finite time breakdown in Eulerian dynamics, \( u_t + u \cdot \nabla_x u = \nabla_x F \). The global behavior is dictated by the different models of the forcing \( F = F(u, \nabla u, \ldots) \). To address these questions, we propose the notion Critical Threshold (CT), where a conditional finite time breakdown depends on whether the initial configuration crosses intrinsic critical surfaces which guarantee global existence. With the standard energy method approach one studies the growth of \( \nabla_x u \). Our approach is based on spectral dynamics, tracing the eigenvalues, \( \lambda := \lambda(\nabla_x u) \), which determine the boundaries of CT surfaces in configuration space.

We demonstrate the CT phenomena with several prototype models. We begin with the \( n \)-dimensional restricted Euler equations, obtaining a surprising \( 4 \)-dimensional global existence for a large set of sub-critical initial data. The second example consists of the corresponding \( n \)-dimensional restricted Euler-Poisson equations. Here we identify a set of \( [n/2] \) spectral invariants, which lead to a remarkable characterization of two-dimensional sub-critical initial configurations with global smooth solutions. Finally, we show how the CT phenomenon associated with rotation prevents finite-time breakdown, which, in turn, yields a long-time regularity regime in the shallow-water equations. Our study reveals the critical dependence of the two-dimensional CT phenomenon on the initial spectral gap, \( \lambda_2(0) - \lambda_1(0) \).