Some New Applications of Conformal Mapping

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Abstract:
Since the nineteenth century, conformal mapping has been used to solve Laplace's equation by exploiting the connection between harmonic and analytic functions. In this talk, we note that another special property of Laplace's equation -- its conformal invariance -- is not unique, but rather is shared by certain systems of nonlinear equations, whose solutions have nothing to do with analytic functions. This simple observation leads to some unexpected applications of conformal mapping in physics. For example, it generates a multitude of exact solutions to the Navier-Stokes equations of fluid mechanics and the Nernst-Planck equations of electrochemical transport. It also allows continuous and stochastic conformal-map dynamics for Laplacian growth (in models of viscous fingering and diffusion-limited aggregation, respectively) to be extended to a broad class of non-Laplacian growth phenomena, such as advection-diffusion-limited aggregation (DLA in a fluid flow). These models provide analytical insights into the effects of competing transport processes and the average shape of fractal clusters.

Monday, December 12, 2005
4:30 PM
Building 2, Room 105

Reception at 4:00 PM in Building 4, Room 174.
(Math Majors Lounge)