Patterns and Symmetries in Networks of Spiking Neurons

Victoria Zhang
Mentor Dr. Bolun Chen, Brandeis University

9th Annual Primes Conference
May 18-19th, 2019
Background

What is a pulsed coupled oscillator?

- Oscillators evolve independently of one another, except when one oscillator reaches a threshold level
- Natural models for both biological and mechanical systems
Introduction to Computational Neuroscience

Background on Neurons

• Surrounded by dissolved ions, which produce a membrane potential

• Excitable, generating action potentials or brief surges in the membrane potential

• When a neuron generates an action potential, the neuron *fires* or *spikes*

Motivation from Computational Neuroscience

• Model oscillations in neural networks as pulsed coupled oscillators

• Simplest form of brain network behavior
Integrate and Fire Model

- for a system with $N$ cells

\[ \dot{x}_i = a - x_i + \sum_{j=1}^{N} K_{ij} s_j(t) \]

- membrane potential $x_i \in [0,1]$ 

- $a > 1$

- coupling strength $K_{ij} \in \mathbb{R}$

- synaptic current $\dot{s}_i = \alpha (-s_i + b_i)$

- auxiliary variable $\dot{b}_i = -\alpha b_i$

in between spikes

\[ \ddot{s}_i + 2\alpha \dot{s}_i + \alpha^2 s_i = 0 \]
Types of Coupling

All-to-all coupling

All-to-all coupling allows us to write:

\[
\dot{x}_i = a - x_i + Ks
\]

in between spikes

\[
\dot{s} = \alpha(-s + b)
\]

\[
\ddot{s} + 2\alpha \dot{s} + \alpha^2 s = 0
\]
Types of Dynamical Solutions (3 Cells)

Cluster Synchrony Solution

Splay State Solution

Splay State near Bifurcation

Limit Cycle Solution
Dynamical Solutions for Three Cells

\[ K = -0.1, \ a = 1.84 \]
Dynamical Solutions for Three Cells

\[ K = -0.1, \ a = 1.84 \]
Dynamical Solutions for Three Cells

$K = 0.1, \ a = 1.84$
Dynamical Solutions for Three Cells

\[ K = 0.1, \ a = 1.84 \]
Preliminary Analytical Results

Parameterize current time as a function of the period: \( t = \theta T \)

\[
x_1(T) = 1 = a(1 - e^{-T}) + e^{-T} \int_0^1 e^{\theta T} s_T(\theta + \phi) d\theta
\]

\[
x_2((1 - \phi)T) = 1 = a(1 - e^{-T}) + e^{-T} T \int_0^1 e^{\theta T} s_T(\theta - \phi) d\theta
\]
Preliminary Analytical Results cont.

\[ G(\phi) = e^{-T} \int_{0}^{1} e^{\theta T} (s_T(\theta + \phi) - s_T(\theta - \phi)) d\theta \]

searching for roots of \( G(\phi) \):
- Search for bifurcation points
- Examine stability

(Van Vreewsijk, Abbott & Ermentrout, 1994)
Future Work

- Extend analytical results to three-cell systems

- Potentially extend the analytical results to even higher-order systems

- Develop the analytical equation for Poincaré map

- Characterize the stability of different dynamical solutions
Acknowledgements

• Dr. Bolun Chen

• My parents

• MIT PRIMES Program
References


