Patterns and Symmetries in Networks of Spiking Neurons

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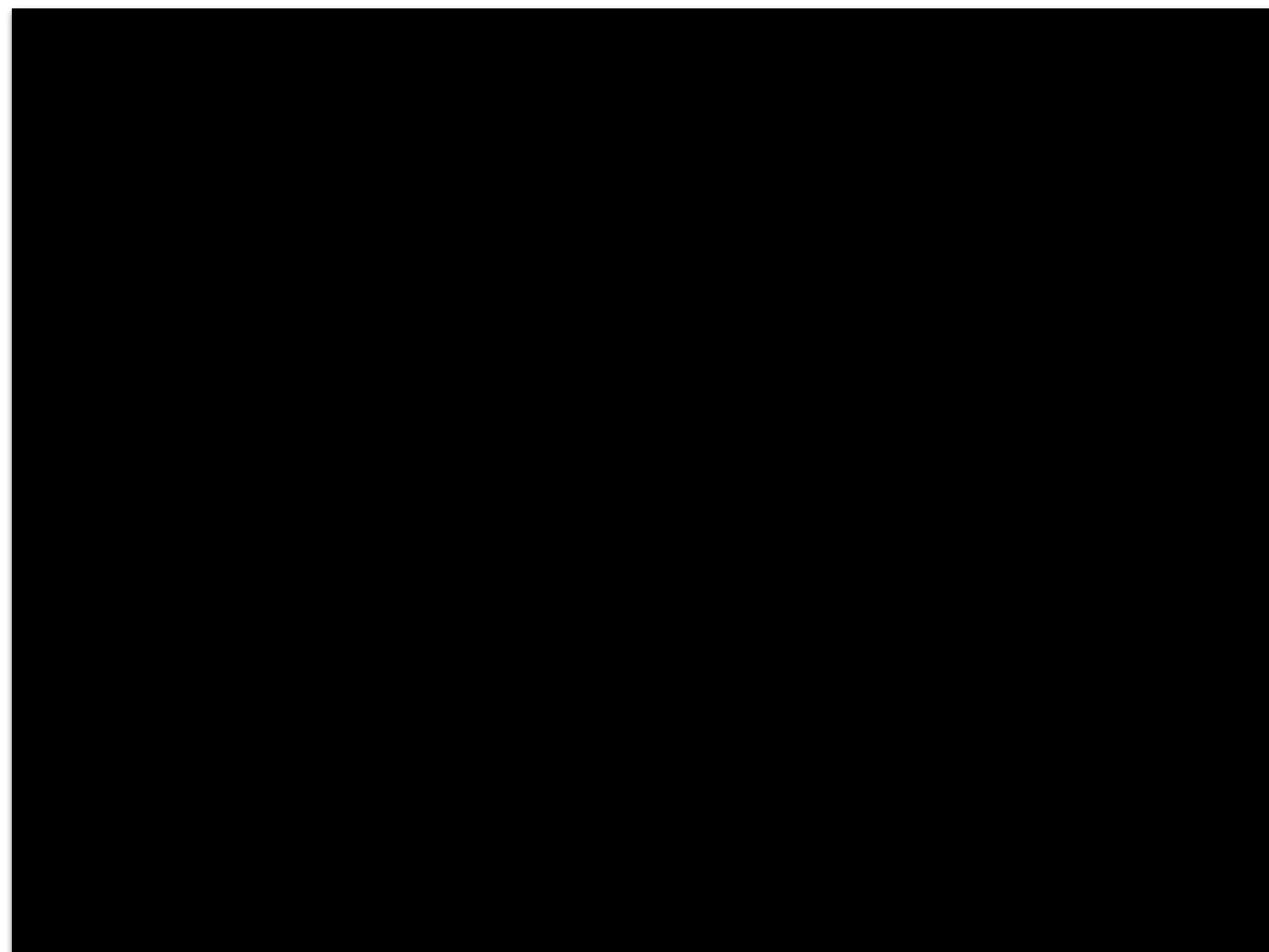
Background

What is a pulsed coupled oscillator?

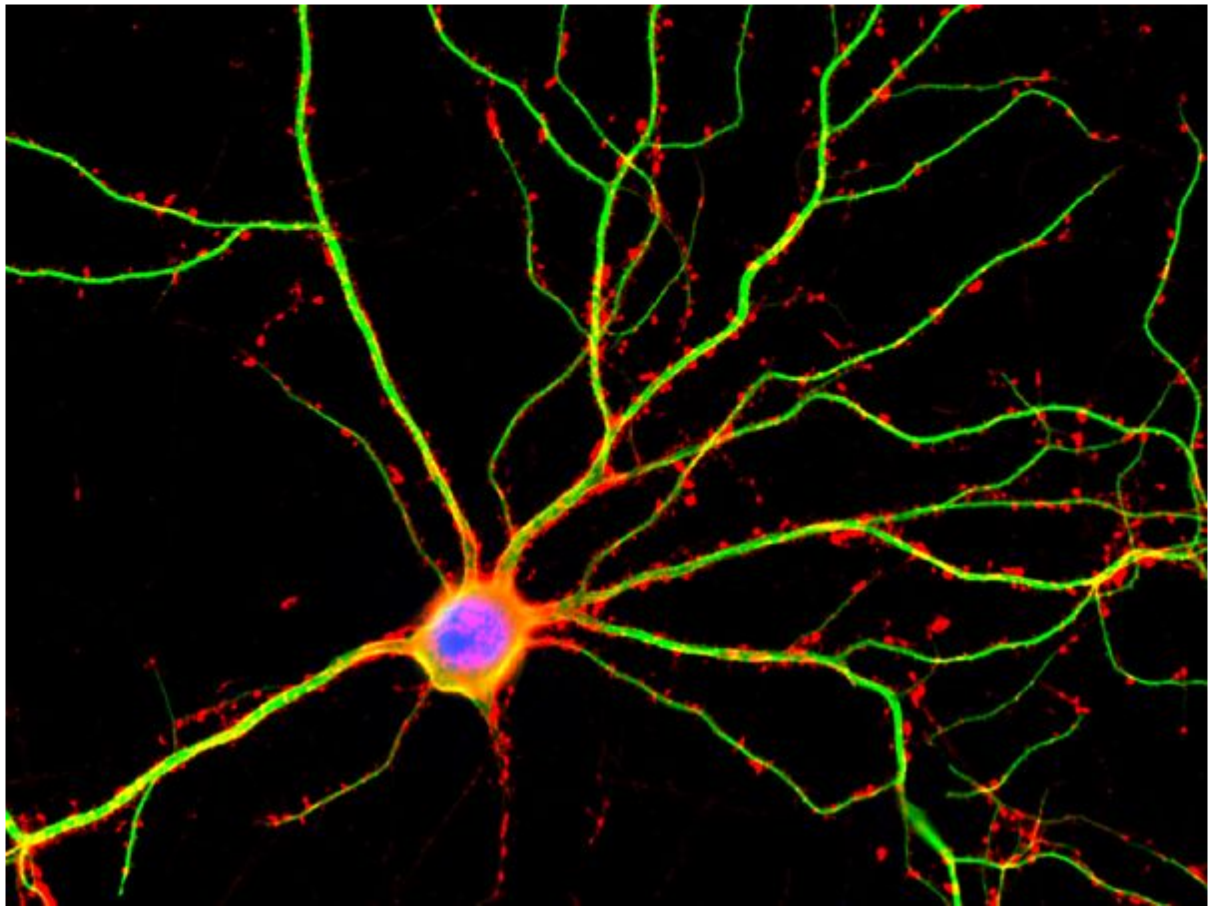
- Oscillators evolve independently of one another, except when one oscillator reaches a threshold level
- Natural models for both biological and mechanical systems

Chicago Tribune



UCLA Department of Physics and Astronomy

Introduction to Computational Neuroscience



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Background on Neurons

- Surrounded by dissolved ions, which produce a membrane potential
- Excitable, generating action potentials or brief surges in the membrane potential
- When a neuron generates an action potential, the neuron *fires* or *spikes*

Motivation from Computational Neuroscience

- Model oscillations in neural networks as pulsed coupled oscillators
- Simplest form of brain network behavior

Integrate and Fire Model

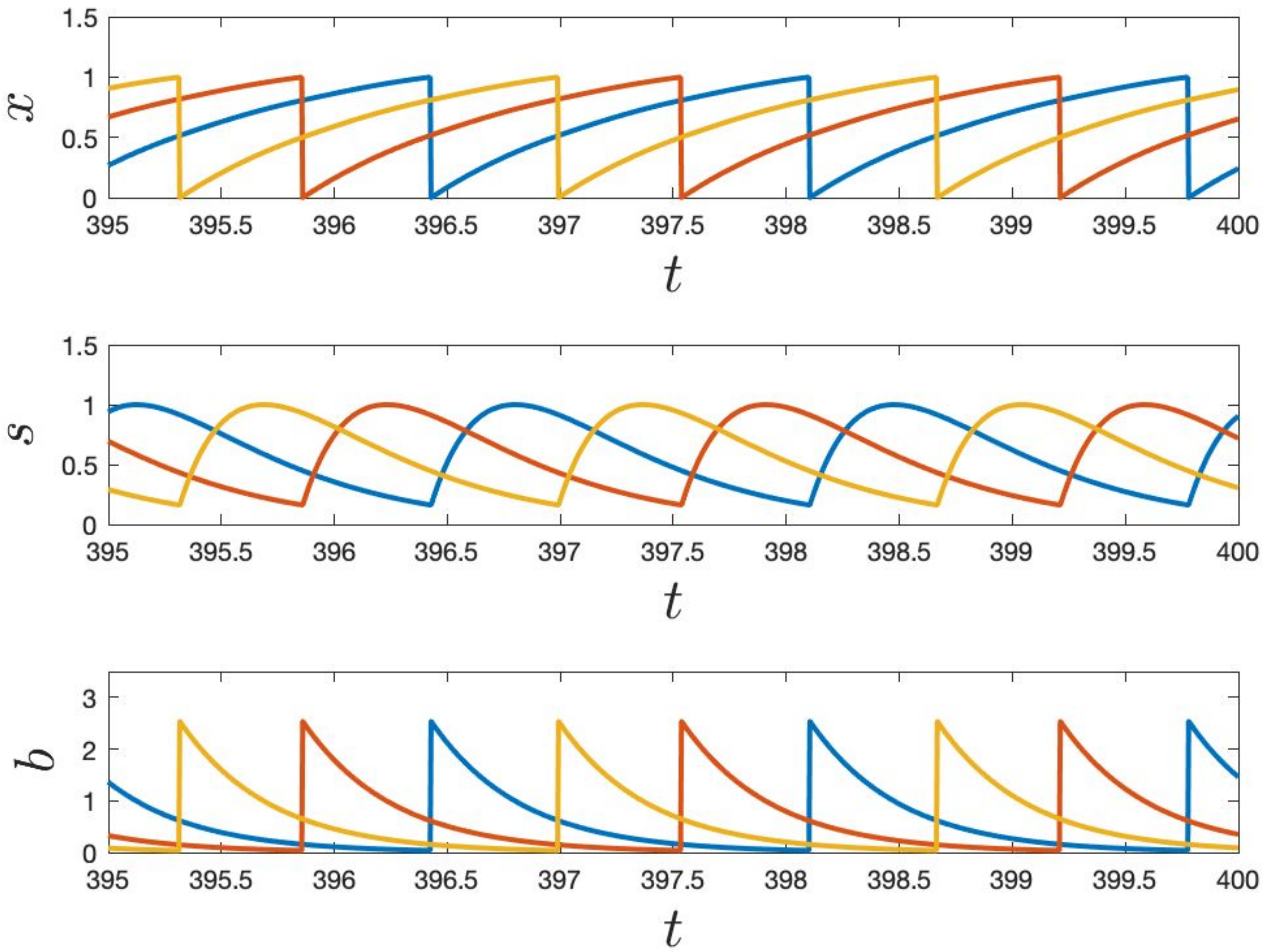
- for a system with N cells

$$\dot{x}_i = a - x_i + \sum_{j=1}^{N} K_{ij} s_j(t)$$

- membrane potential $x_i \in [0,1]$
- $a > 1$
- coupling strength $K_{ij} \in \mathbb{R}$
- synaptic current $\dot{s}_i = \alpha(-s_i + b_i)$
- auxiliary variable $\dot{b}_i = -\alpha b_i$

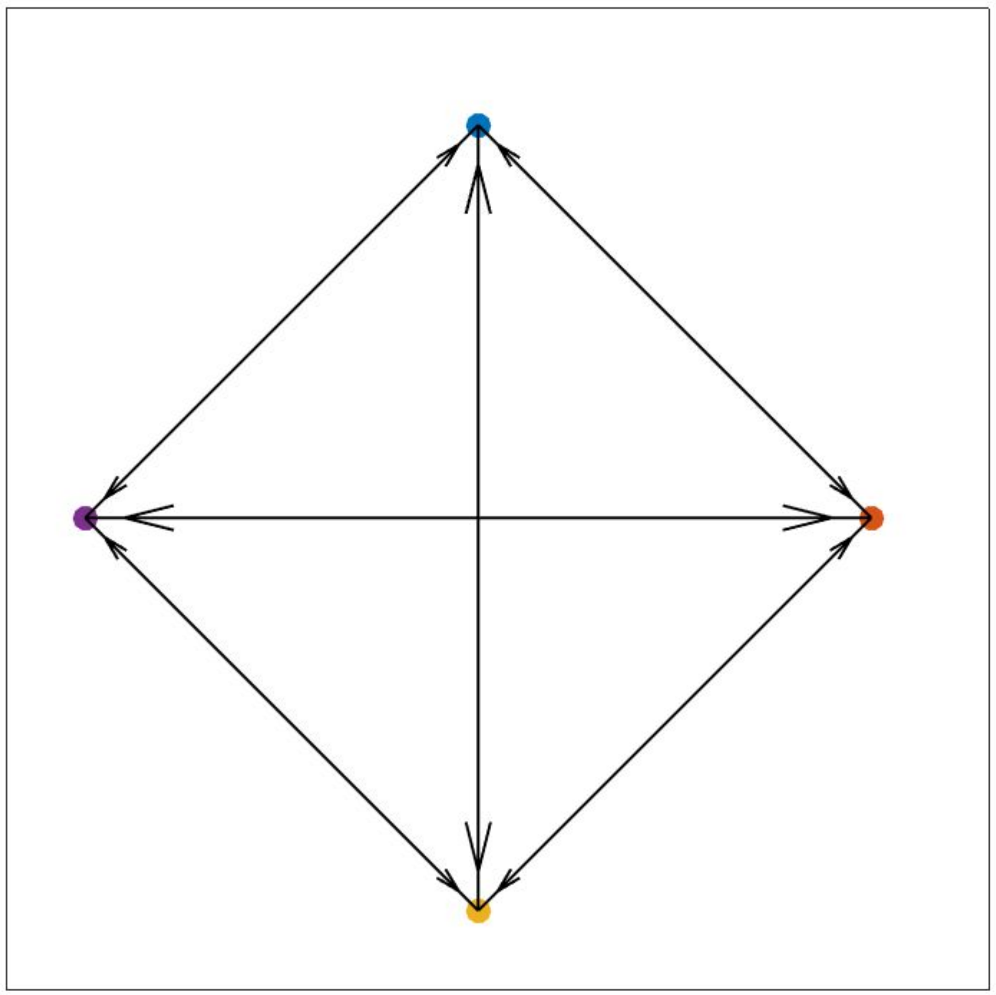
in between spikes

$$\ddot{s}_i + 2\alpha \dot{s}_i + \alpha^2 s_i = 0$$

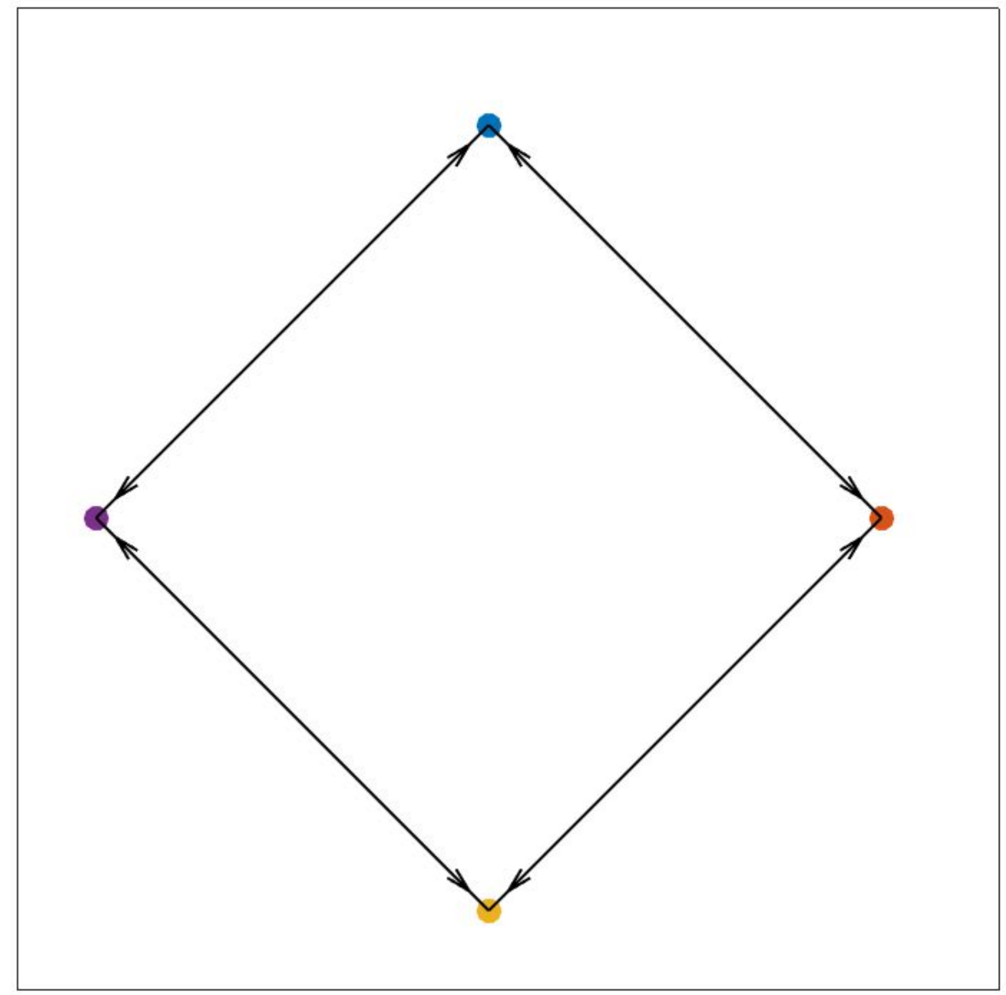


Types of Coupling

All-to-all coupling



Neighboring coupling



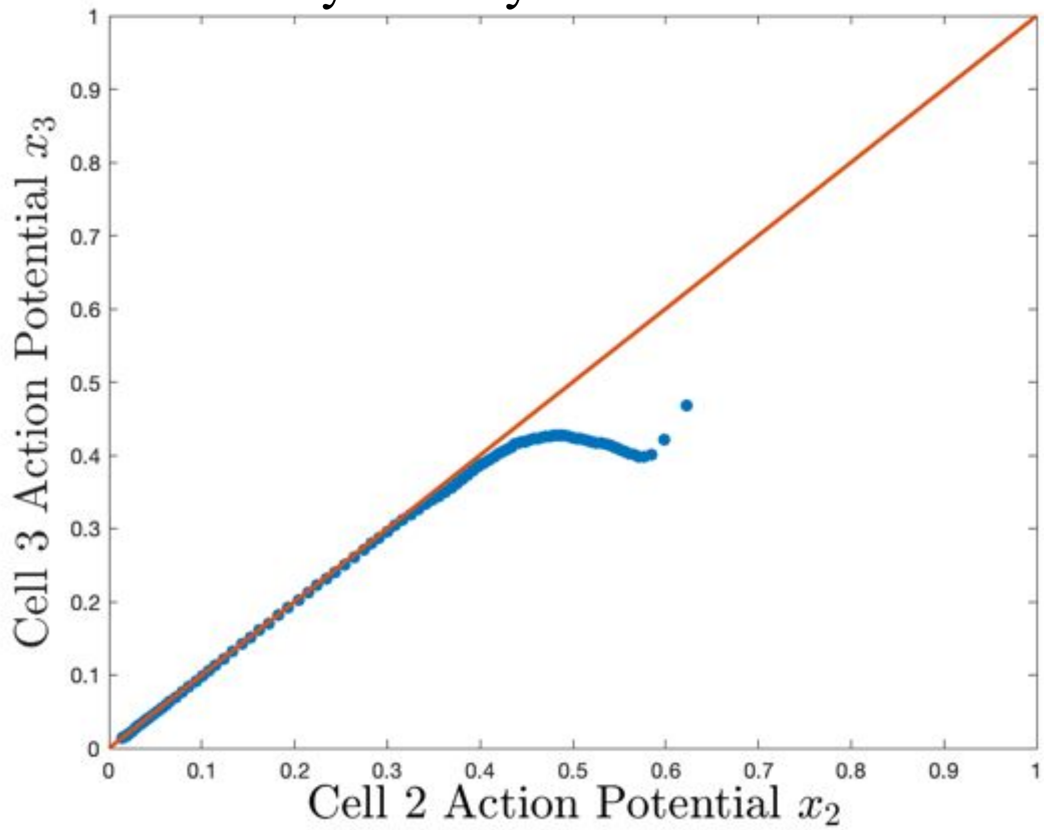
All-to-all coupling allows us to write:

$$\dot{x}_i = a - x_i + Ks \qquad \dot{s} = \alpha(-s + b) \qquad \dot{b} = -\alpha b$$

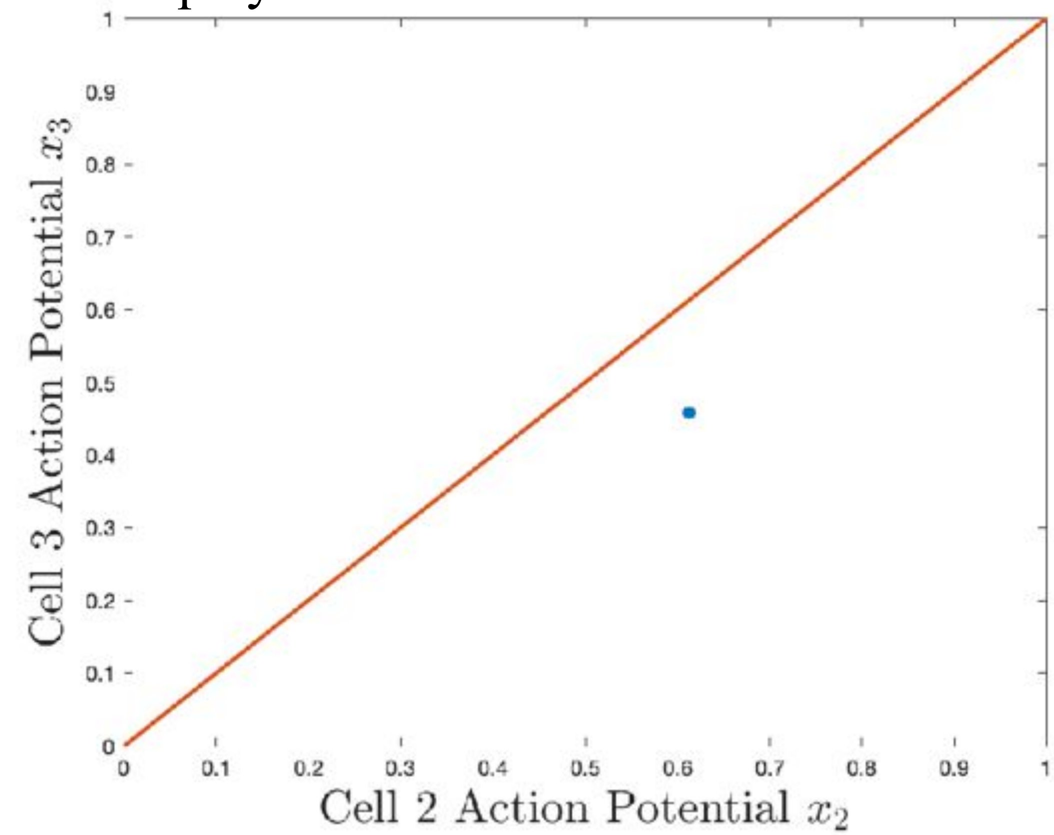
in between spikes $$\ddot{s} + 2\alpha\dot{s} + \alpha^2 s = 0$$

Types of Dynamical Solutions (3 Cells)

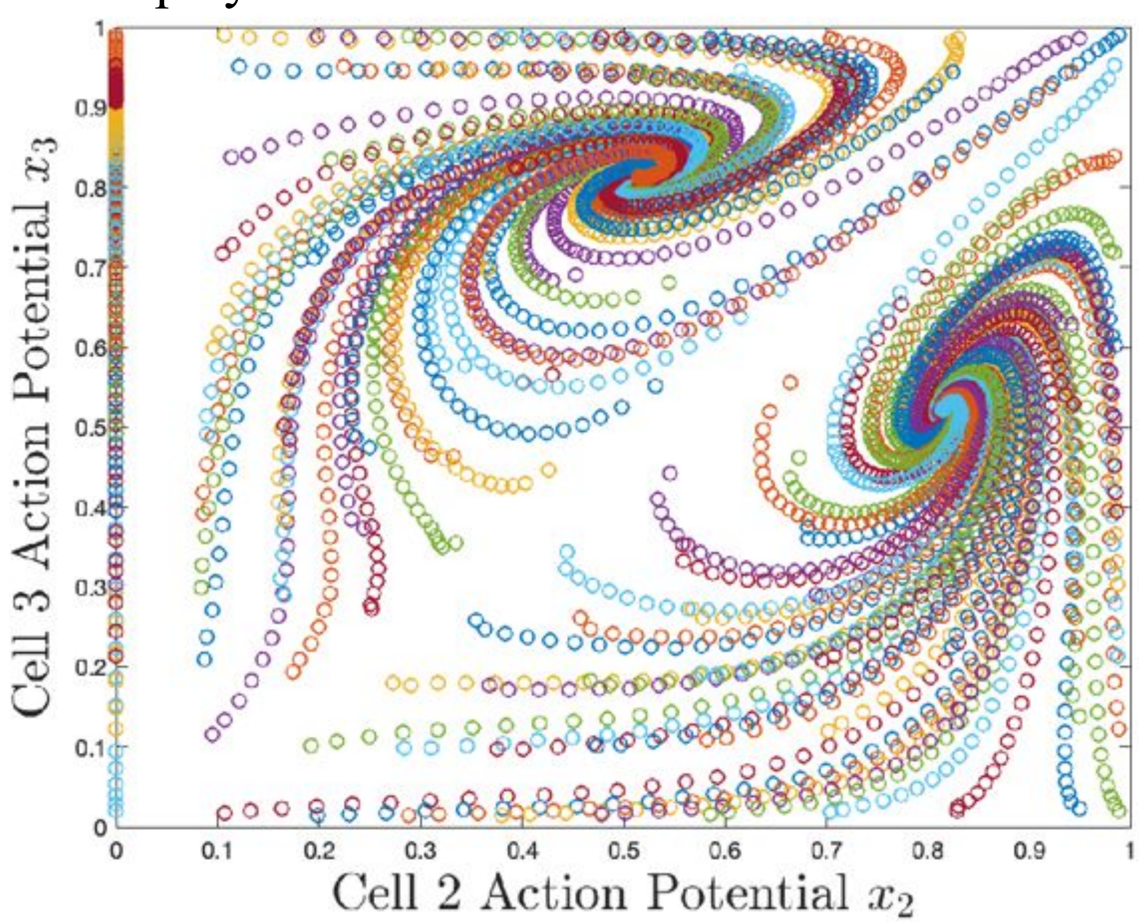
Cluster Synchrony Solution



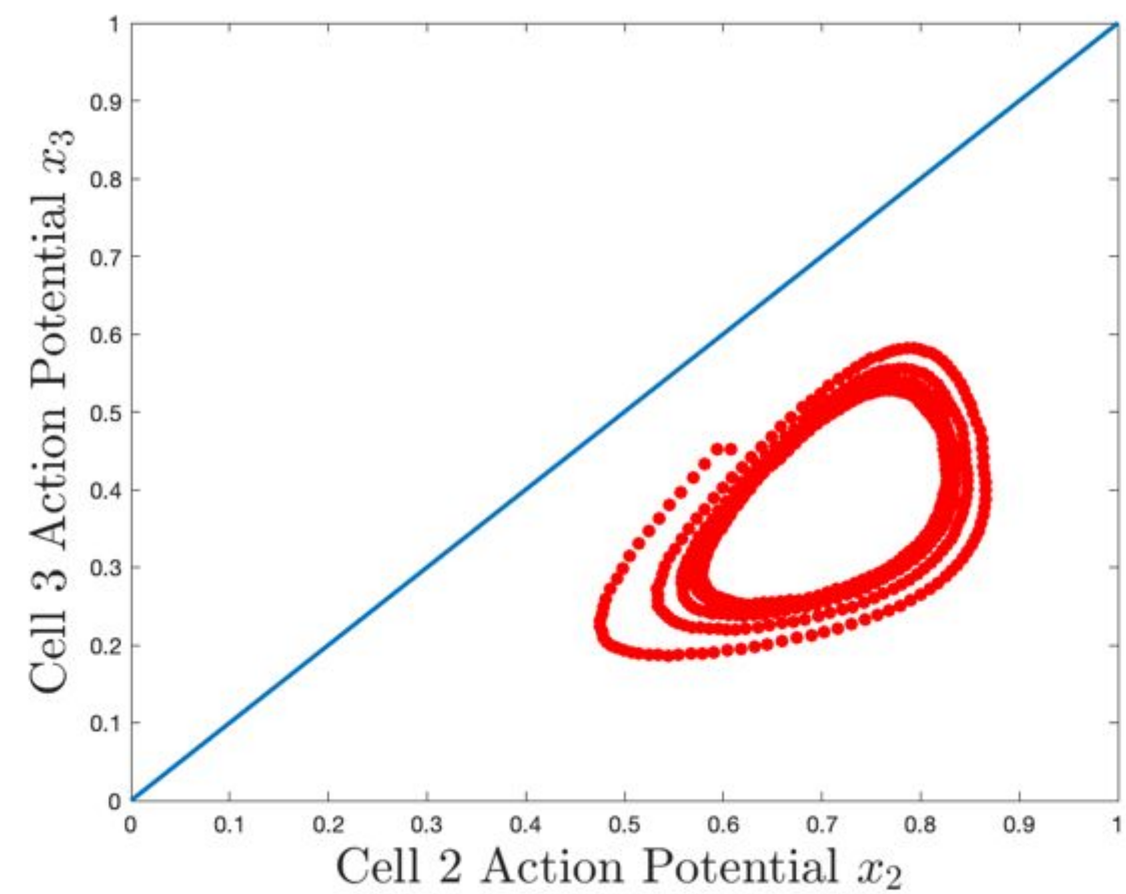
Splay State Solution



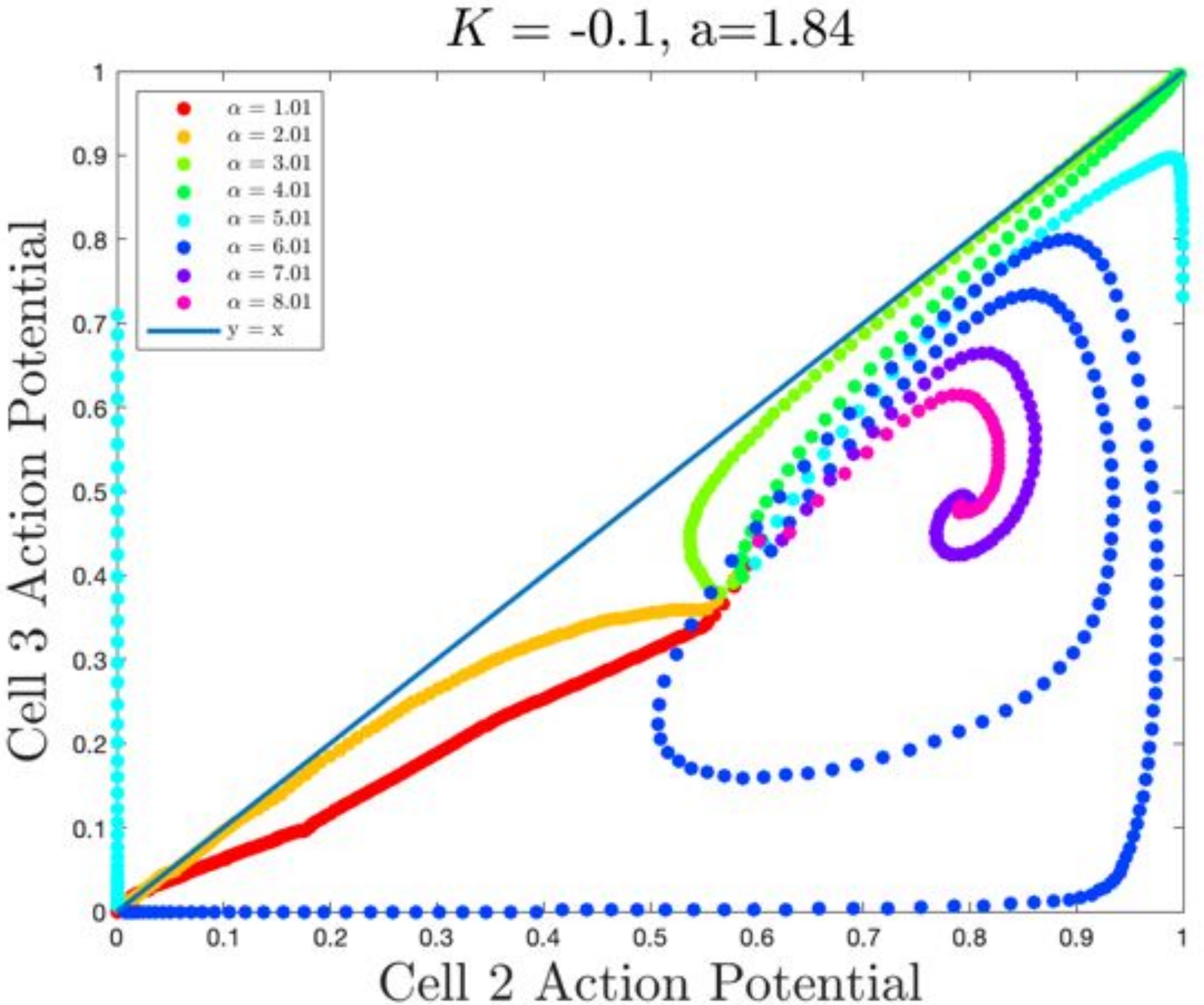
Splay State near Bifurcation



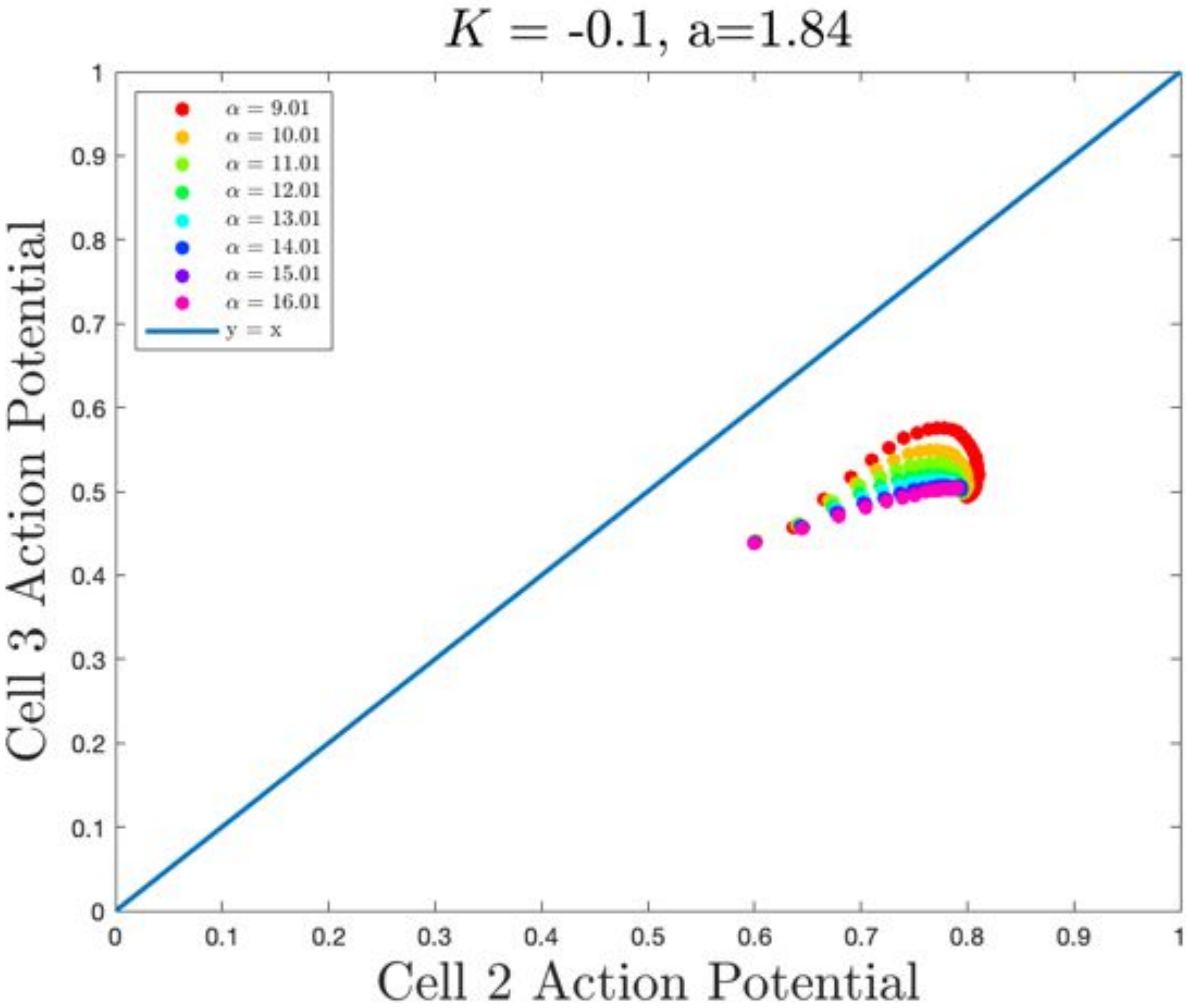
Limit Cycle Solution



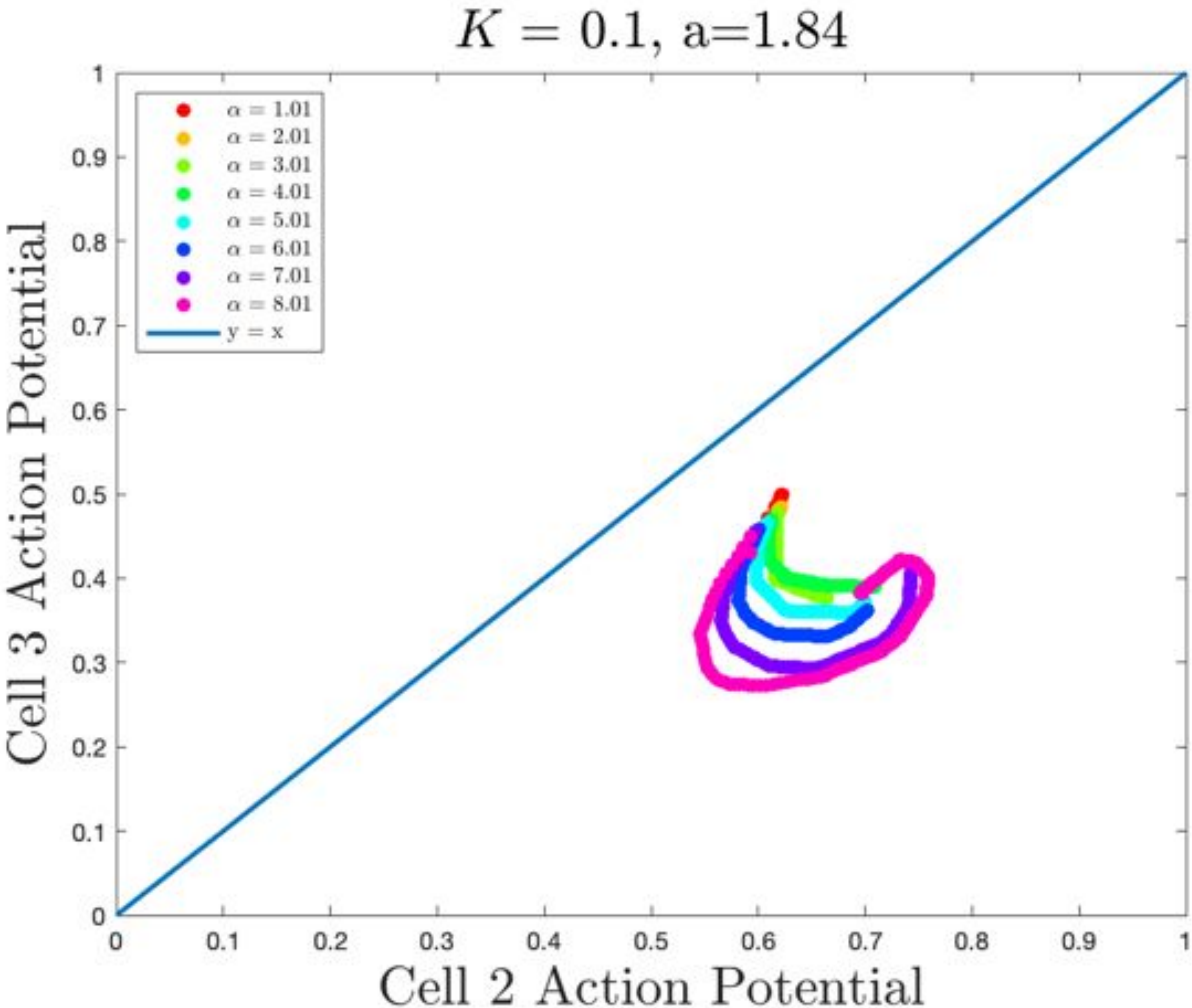
Dynamical Solutions for Three Cells



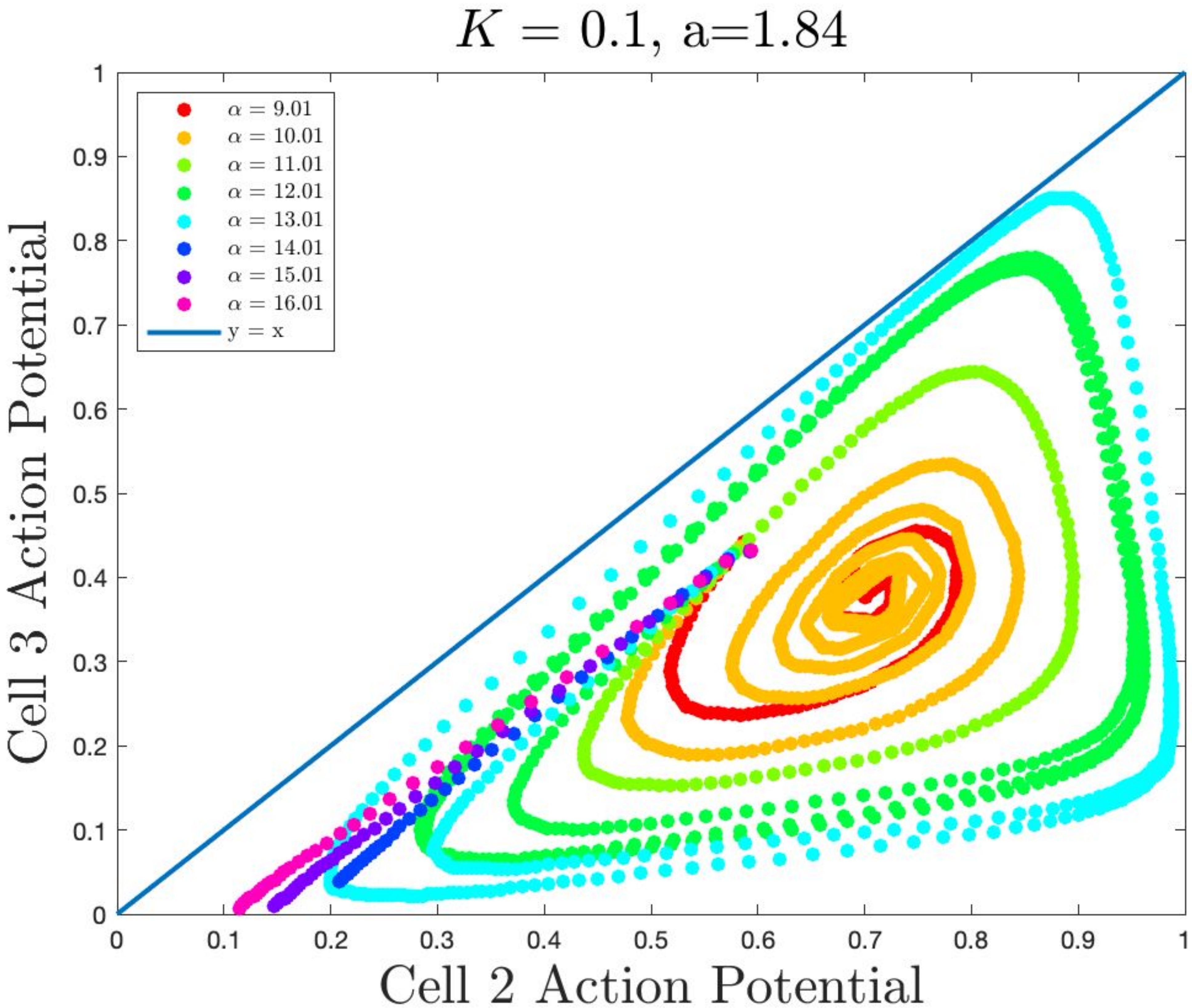
Dynamical Solutions for Three Cells



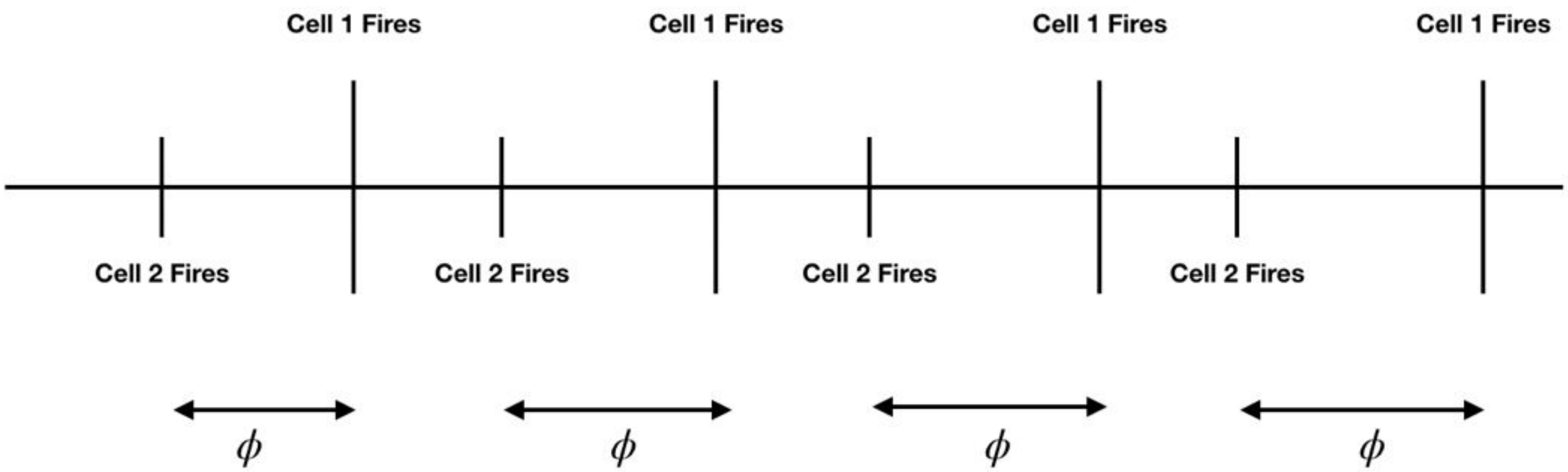
Dynamical Solutions for Three Cells



Dynamical Solutions for Three Cells



Preliminary Analytical Results

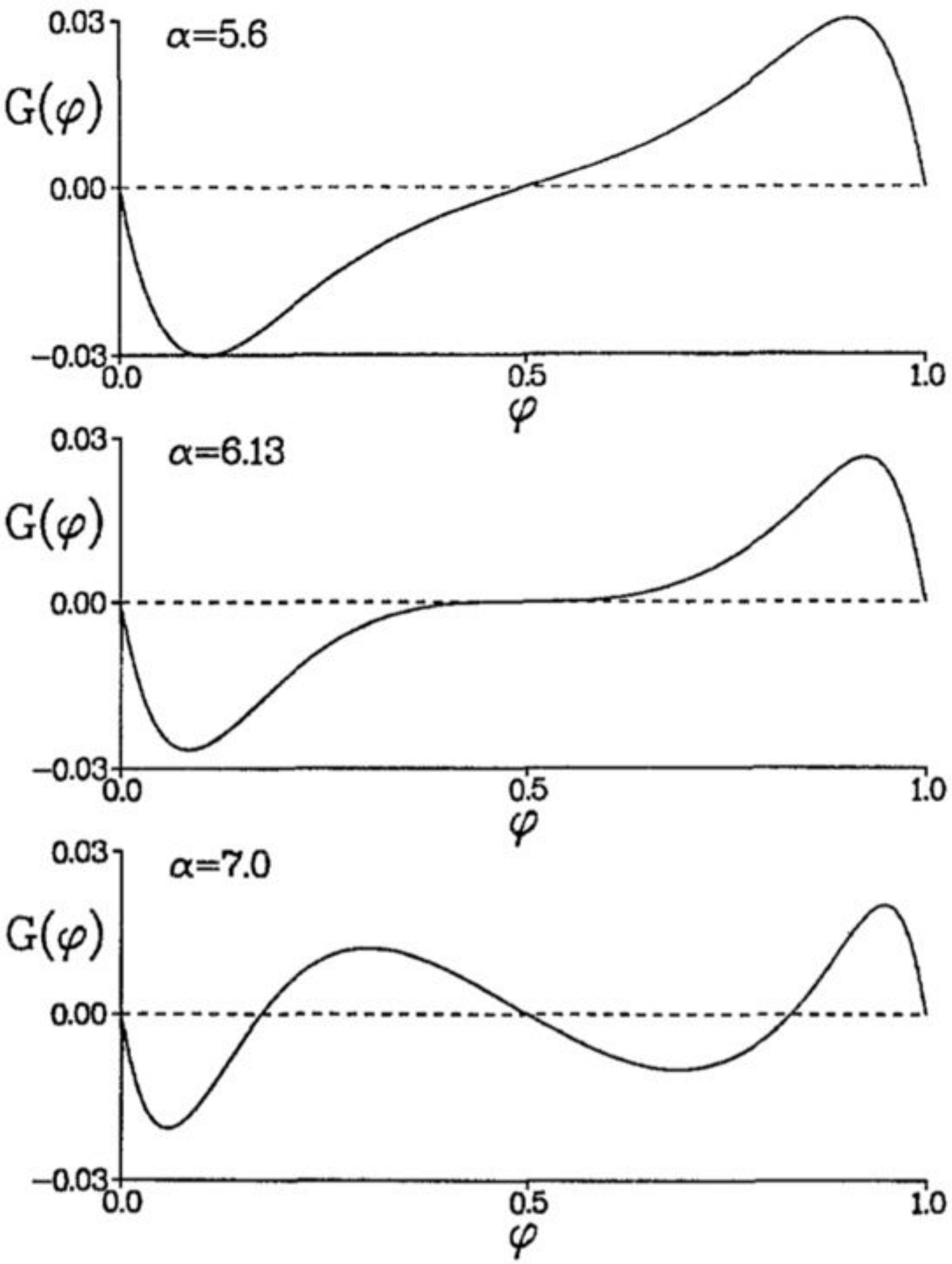


Parameterize current time as a function of the period: $t = \theta T$

$$x_1(T) = 1 = a(1 - e^{-T}) + e^{-T} \int_0^1 e^{\theta T} s_T(\theta + \phi) d\theta \qquad (1)$$

$$x_2((1-\phi)T) = 1 = a(1 - e^{-T}) + e^{-T} T \int_0^1 e^{\theta T} s_T(\theta - \phi) d\theta \qquad (2)$$

Preliminary Analytical Results cont.

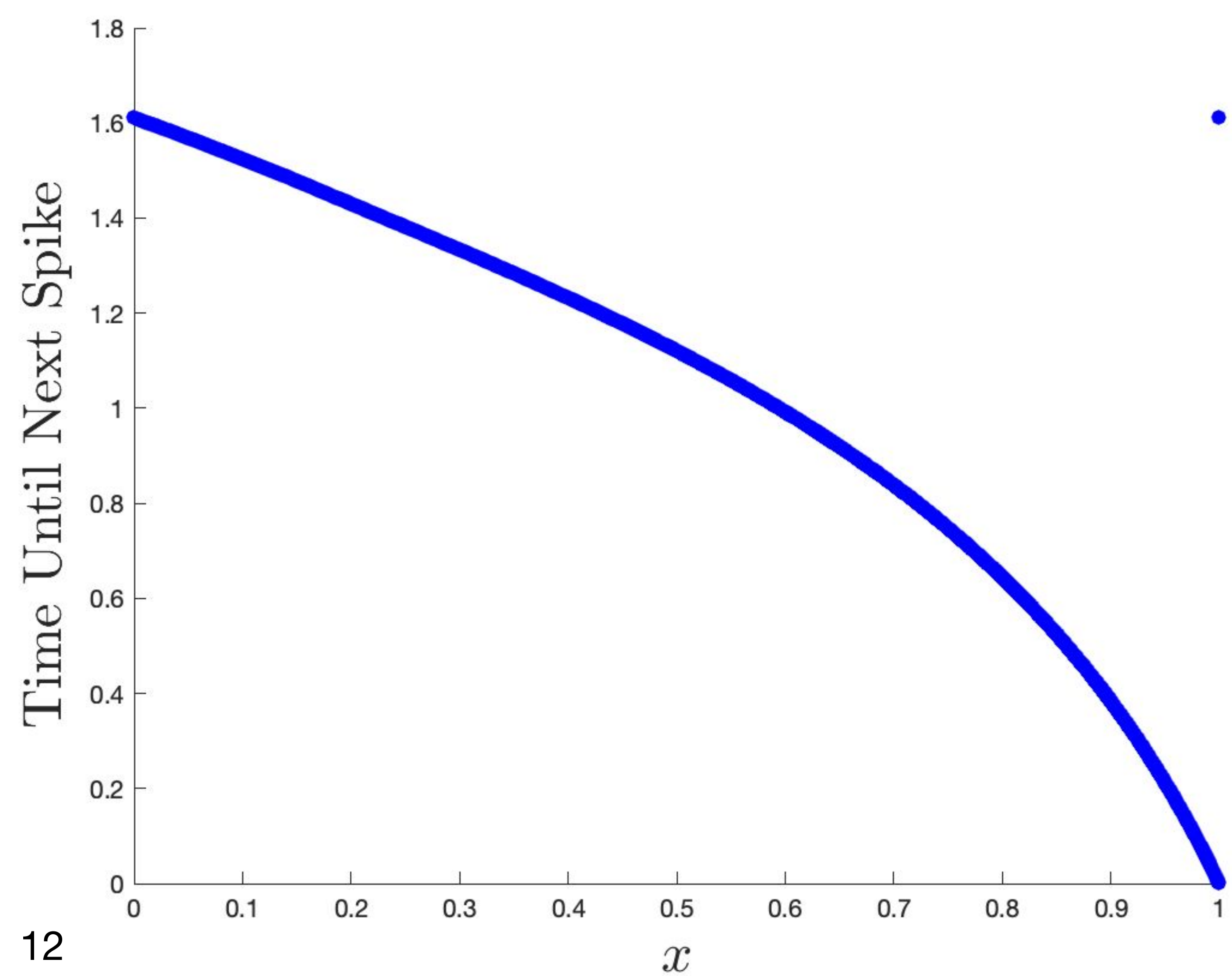


(Van Vreewsijk, Abbott & Ermentrout, 1994)

$$G(\phi) = e^{-T}\int_0^1 e^{\theta T}(s_T(\theta+\phi) - s_T(\theta-\phi))d\theta$$

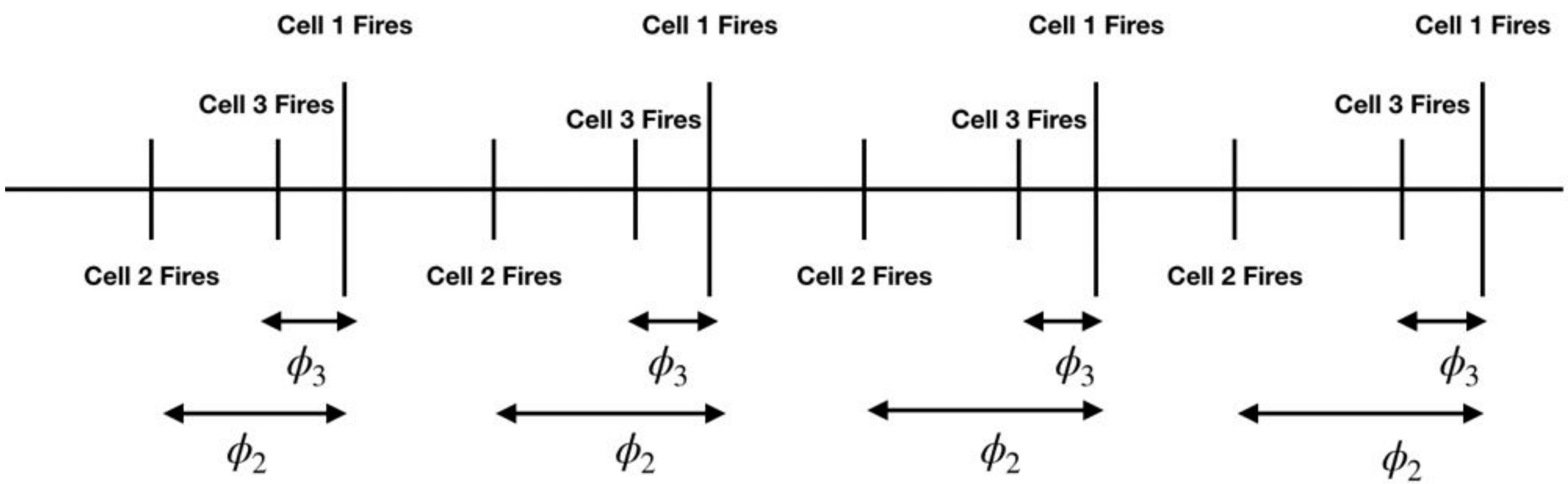
searching for roots of $G(\phi)$:

- Search for bifurcation points
- Examine stability



Future Work

- Extend analytical results to three-cell systems



- Potentially extend the analytical results to even higher-order systems

- Develop the analytical equation for Poincaré map

- Characterize the stability of different dynamical solutions

Acknowledgements

- Dr. Bolun Chen

- My parents

- MIT PRIMES Program

References

- [1] Renato E. Mirollo and Steven H. Strogatz. Synchronization of pulse-coupled biological oscillators. *SIAM Journal on Applied Mathematics*, 50(6):1645–1662, 1990.
- [2] Bolun Chen, Jan R. Engelbrecht, and Renato Mirollo. Cluster synchronization in networks of identical oscillators with α-function pulse coupling. *Phys. Rev. E*, 95:022207, Feb 2017.
- [3] Simona Olmi, Antonio Politi, and Alessandro Torcini. Stability of the splay state in networks of pulse-coupled neurons. *The Journal of Mathematical Neuroscience*, 2(1):12, Nov 2012.
- [4] Carl Van Vreeswijk, L. F. Abbott, and G. Bard Ermentrout. When inhibition not excitation synchronizes neural firing. *Journal of Computational Neuroscience*, 1(4):313–321, Dec 1994.
- [5] L. F. Abbott and Carl van Vreeswijk. Asynchronous states in networks of pulse-coupled oscillators. *Phys. Rev. E*, 48:1483–1490, Aug 1993.