ELLIPITC KRV

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THE ASSOCIATIVE ALGEBRA

\[ x \rightarrow y \rightarrow y \rightarrow x \rightarrow x \rightarrow x \rightarrow y \]

corresponds to

\[ xyyxxxxy \quad (\neq xxxyyyy) \]
THE ROOTED LIE TREE

\[ y, x \]

\[ y \]

\[ x \]

\[ y = [y, x] \]

\[ y = [y, [x, y]] \]

\[ [[[y, x], y] \neq [y, [x, y]]] \]
THE LIE TREE, or F(L)

corresponds to

a derivation:

\[ u(x) = 2[x, [y, x]] \]

\[ u(y) = -2[[y, x], y] \]
The Lie Brackets $[\cdot, \cdot]$ gives a bilinear operation that satisfies:

1. (Antisymmetry)
   
   $$[x, y] = -[y, x]$$

2. (Jacobi Identity)
   
   $$[x, [y, z]] + [z, [x, y]] + [y, [z, x]] = 0$$
The Lie brackets \([ \cdot, \cdot \] \) gives a bilinear operation that satisfies:

1. **(Antisymmetry)**
   \[[x, y] = -[y, x]\]

2. **(Jacobi Identity)**
   \[[x, [y, z]] + [z, [x, y]] + [y, [z, x]] = 0\]
**DEF: krv**

$$\text{k}rv = \text{k}rv^{(1,1)} = \{ u \in \text{tder}(1,1) = \text{Der}^+(L(x,y)) \mid u([x,y]) = 0, \text{div}(u) = 0 \}$$

We’ll understand the definition in terms of **graphs**.
\[ \text{Grv} = \{ \text{der } u \mid u([x, y]) = 0, \ \text{div}(u) = 0 \} \]

**Graphical Interpretation of** \( u([x, y]) = 0 \)

Recall:

\[
\begin{align*}
&u(x, y) = 0 \\
&u(x) = 2[x, [y, x]] \\
&u(y) = -2[[y, x], y]
\end{align*}
\]
THE TRACE

Recall that \( x - y - y - x - x \)
is a term in the Associative Algebra

\[
\text{tr}(x - y - y - x - x) = \begin{array} \end{array}
\]
\[ \ker v = \{ \text{der } u \mid u([x, y]) = 0, \ \text{div}(u) = 0 \} \]

**Graphical Interpretation of div**

Graphs representing the conditions for \( \ker v \).
$\ker v = \{ \text{der } u | u([x, y]) = 0, \text{ div}(u) = 0 \}$

Graphical Interpretation of $\text{div}$

GRAPHICAL REPRESENTATIONS
$\kappa_{\nu} = \{ \text{der } u | u([x, y]) = 0, \text{ div}(u) = 0 \}$

**Graphical Interpretation of div**

\[ \begin{align*}
    \text{div} &= \text{div}(u) = 0 \\
    &= \text{div}(u) = 0 \\
    &= \text{div}(u) = 0 \\
    &= \text{div}(u) = 0 \\
    &= \text{div}(u) = 0
\end{align*} \]
CURRENT PROGRESS

Def: the birds-on-a-wire graph

Claim:

Every lie tree can be rewritten as birds-on-a-wire graph
PROGRESS AND GOALS

BASIS FOR \( F(L)^{\leq 7} \)

\[
\begin{align*}
x - x & x - y & y - y & x \quad | \quad x & y & y & x \\
x & y & y & y & y & x & x & x & x & y & y & y & y & x
\end{align*}
\]

GRAPHICAL REPRESENTATIONS

\[
\begin{align*}
x - y - y - x - x
\end{align*}
\]
PROGRESS AND GOALS

MORE SMALL ELEMENTS

- Elements in krv with 2 $x$’s

- Elements in krv with 3 $x$’s (?)
GOALS

- Study the elements of $\kappa\nu$ with small total number of $x$ and $y$’s.
- Possibly using a computer
- Study the elements of $\kappa\nu$ with small number of $x$’s.
  Currently, we are working on elements with 3 $x$’s.
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