(Photo Credit: Slava Gerovitch)

Eighth Annual Conference
May 2018

Eighth Annual PRIMES Conference

May 19–20, 2018

Saturday, May 19

8:30 am **<u>Welcoming Remarks</u>**

- Prof. Michel Goemans, Head of the MIT Mathematics Department
- Prof. Pavel Etingof, PRIMES Chief Research Advisor
- Dr. Slava Gerovitch, PRIMES Program Director

9:00 am **<u>Session 1: Combinatorics</u>**

- James Lin, Carl Joshua Quines, and Espen Slettnes, *Throttling numbers for Cop vs Gambl*er (mentor Dr. Jesse Geneson, Penn State)
- Nathan Ramesh, *Maximal difference avoiding subsets of Z* (mentor Christian Gaetz)
- Sean Elliott, *Anti-Ramsey type problems* (mentor Dr. Asaf Ferber)
- Ezra Erives, Srinivasan Sathiamurthy, and Theodore Baas, *Asymptotics of visibility in three-dimensional grid worlds* (mentor Dr. Zarathustra Brady)

10:35 am **<u>Session 2: Geometry and Topology</u>**

- Ryan Kim, *Cohomology groups of the dual Steenrod algebra* (mentor Sanath Devalapurkar)
- Daniel Liu, *New fusion products for the affine sl(2) at level k=1/2* (mentor Dr. Claude Eicher)
- Jiwon Choi, *Gromov-Hausdorff distance between metric spaces* (mentor Ao Sun)
- Sathwik Karnik, *Bounds on the maximal cardinality of an acute set in a hypercube* (mentor Ao Sun)

11:50 am **<u>Session 3: Knot Theory</u>**

- Max Guo, *On Khovanov homology, Bar-Natan's perturbation, and Conway mutation* (mentor Dr. Jianfeng Lin)
- Aayush Karan, *Mutation invariance of the Szabó spectral sequence* (mentor Dr. Jianfeng Lin)
- Nithin Kavi and Wendy Wu, *Width and trunk of satellite knots* (mentor Zhenkun Li)

2:00 pm **<u>Session 4: PRIMES Circle</u>**

- Isabel Vogt, PRIMES Circle Coordinator, *Introduction*
- Sherry Lim and Mirilla Zhu, *Permutations for preschoolers: Applications of group theory to childhood games* (mentor Margalit Glasgow)
- Sekai Carr and Laura Clervil, *Connect Spokes: An original combinatorial game* (mentor Marisa Gaetz)
- Elizabeth Euwart and Anna Rasmussen, *Graphs and the chromatic polynomial* (mentor Dhruv Ranganathan)
- Carly Maggiolo Cabrera and Elisandra Fontes, *Probability, statistical tests, and fair die* (mentor Alexis Oriole)

3:40 pm **<u>Session 5: Applied Mathematics</u>**

- Rinni Bhansali, *Modelling epidemics on polluted networks and* m-*state networks* (mentor Prof. Laura Schaposnik, University of Illinois at Chicago)
- Vincent Huang, *Mathematical and algorithmic models of refugee crises* (mentor Prof. James Unwin, University of Illinois at Chicago)
- Kaiying Hou, *Agent-based models for conservation equations* (mentor Andrew Rzeznik)

4:35 pm **<u>Session 6: Algebraic Combinatorics and Probability</u>**

- Melinda Sun, *Counting bimonotone subdivisions* (mentor Dr. Elina Robeva)
- Haneul Shin, *Bimonotone subdivisions in high dimensions* (mentor Dr. Elina Robeva)
- Stanley Wang, *Moduli space of planar tropical curves of genus 1* (mentor Yu Zhao)
- Gopal Goel and Andrew Yao, *Derivatives of the Gaussian free field via random matrix theory* (mentor Andrew Ahn)

5:50 pm **<u>Session 7: Representation Theory and Number Theory</u>**

- Yuting (Emma) Qin, *The elliptic Kashiwara-Vergne Lie algebra* (mentor Dr. Florian Naef)
- Byung Yeon Rhee, *Elliptic curves, factorization, and cryptography* (mentor Yongyi Chen)
- Merrick Cai, *The Hilbert series of the irreducible representation of the rational Cherednik algebra of type A_n in characteristic* p (mentor Daniil Kalinov)
- Archer Wang, *Hilbert series of quasi-invariant polynomials* (mentor Dr. Xiaomeng Xu)

Sunday, May 20

8:45 am <u>Welcoming Remarks</u>

- Dr. Slava Gerovitch, PRIMES Program Director
- Dr. Tanya Khovanova, PRIMES Head Mentor

9:00 pm <u>Session 8: Combinatorics</u>

- Vinjai Vale, *A new paradigm for computer vision based on compositional representation* (mentor Kevin Ellis)
- Ben Chen, Richard Chen, Joshua Guo, Shane Lee, Neil Malur, Nastia Polina, Poonam Sahoo, Anuj Sakarda, Nathan Sheffield, and Armaan Tipirneni (PRIMES STEP Senior students), *On base 3/2 and its sequences* (mentor Dr. Tanya Khovanova)
- Matvey Borodin, Hannah Han, Kaylee Ji, Alexander Peng, David Sun, Isabel Tu, Jason Yang, William Yang, Kevin Zhang, and Kevin Zhao (PRIMES STEP Junior students), *Chips Go BOOM BOOM!!!* (mentor Dr. Tanya Khovanova)

10:10 am <u>Session 9: Combinatorics</u>

- Vincent Bian, *Packing anchored rectangles* (mentor Dr. Tanya Khovanova)
- Eric Zhang, *On quasirandom permutations* (mentor Dr. Tanya Khovanova)
- Wayne Zhao, *Counting Sudoku variants* (mentor Dr. Tanya Khovanova)

11:05 am <u>Session 10: Analysis</u>

- William Fisher, *Polynomial Wolff axioms and Kakeya-type estimates for bent tubes* (mentor Robert Burklund)
- Alan Yan, *Asymptotic counting in dynamical systems* (mentor Prof. Sergiy Merenkov, CCNY – CUNY)
- Andy Xu, *Approximating the Hurwitz zeta function* (mentor Hyun Jong Kim)

12:00 pm <u>Session 11</u>

- Aurash Vatan, *Finding generators of Jacobian groups of finite graphs* (mentor Dr. Xiaomeng Xu)
- Daniel Zhu, *On the Okounkov-Olshanski formula for the number of tableaux of skew shapes* (mentor Prof. Alejandro Morales, UMass Amherst)
- Allen Wang, *Permutations with up-down signatures of nonnegative partial sums* (mentor Guangyi Yue)

2:00 pm **Welcoming Remarks**

- Prof. Srini Devadas, Department of Electrical Engineering and Computer Science

2:10 pm **Session 12: Computer Science**

- Vivek Bhupatiraju, *accAAD: Efficient Append-Only Authenticated Dictionary for Transparency Logs* (mentor Alin Tomescu)
- Theodor Lukin Yelin, *Designing private forums using differential privacy* (mentor Albert Kwon)
- Sanjit Bhat and David Lu, *Var-CNN and DynaFlow: Improved attacks and defenses for website fingerprinting* (mentor Albert Kwon)
- Harshal Sheth and Andrew Sun, *Extending distributed-systems tracing vertically into the Linux kernel* (mentor Dr. Raja Sambasivan, Boston University)

3:30 pm **Session 13: Medical Informatics**

- Makiah Bennett, *Lexical AI for patient-centered clinical diagnoses* (mentor Prof. Gil Alterovitz)
- Andrew Gritsevskiy, *Capsule networks for low-data transfer learning* (mentors Prof. Gil Alterovitz and Maxim Korablyov)
- Kevin Hu, *Discovery of exon splicing relationships across cancer cell lines* (mentor Dr. Mahmoud Ghandi, Broad Institute)

4:25 pm **Session 14: Math Reading Groups**

- Nhat Pham and Anmol Sakarda, *Concrete mathematics: Exploring summations* (mentor Zhulin Li)
- Dylan Pentland, *The* j-*invariant of an elliptic curve* (mentor Chun Hong Lo)

5:10 pm **Session 15: PRIMES Circle**

- Aneesha Manne and Lara Zeng, *Generating functions in combinatorics* (mentor Uma Roy)
- Iris Yang and Victoria Zhang, *Gaussian integers and their relationship to ordinary integers* (mentor Matthew Weiss)
- Sam Costa and Elin Gu, *Inverse knots and amphicheirality* (mentor Joseph Zurier)

SATURDAY, MAY 19
MATHEMATICS

SESSION 1: COMBINATORICS

James Lin, Carl Joshua Quines, and Espen Slettnes

Throttling Numbers for Cop vs Gambler

Mentor: Dr. Jesse Geneson, Penn State

Project suggested by Dr. Jesse Geneson

We consider a variation of the cop and robber game called the cop and gambler game. There are two players, a cop and a gambler. At time $t = 0$, the cop chooses an initial vertex, then the gambler chooses a probability distribution over the vertices and goes to a vertex chosen by it. At all subsequent times until the end of the game, the cop either moves to an adjacent vertex or stays put and the gambler goes to a vertex chosen by his probability distribution. The game ends when the cop and gambler are at the same vertex. The cop wants to minimize the expected capture time (the value of t when the game ends), whereas the gambler wants to maximize it.

This game can be adapted to k or more cops, where two cops can occupy the same vertex, one cop is needed to catch the gambler, and all cops can communicate with each other. If the cops know the distribution chosen by the gambler, the gambler is said to be known, otherwise it is said to be unknown. The (un)known gambler throttling number of a graph G is the minimum possible sum over all positive integers k of the expected capture time that k cops can achieve against the (un)known gambler and k. We find $\theta(\sqrt{n})$ bounds on the (un)known gambler throttling numbers of various graphs on n vertices.

Nathan Ramesh

Maximal Difference Avoiding Subsets of $\mathbb{Z}$

Mentor: Christian Gaetz

Project suggested by Prof. James Propp, UMass Lowell

Let D be a finite set of positive integers. Call a subset S of $\mathbb{N}$ or $\mathbb{Z}$ *D-avoiding* if there do not exist $x, y \in S$ such that $x - y \in D$. In this talk, we explore properties of maximal D-avoiding subsets of $\mathbb{Z}$ and their connection to the lonely runner conjecture.

Sean Elliott

Anti-Ramsey Type Problems

Mentor: Dr. Asaf Ferber

Project suggested by Dr. Asaf Ferber

A classical theorem in graph Ramsey theory says the following: Given a finite number of colors and a positive integer p, any edge-coloring of the complete graph K_n will contain a monochromatic copy of K_p as long as n is sufficiently large. An important problem is to find the minimum possible such n. We will discuss a class of problems which generalizes this problem. For example, consider a coloring of K_n in which every copy of K_4 uses at least 3 distinct colors. For a given n, what is the minimum number of colors that can be used to produce such a coloring?

Ezra Erives, Srinivasan Sathiamurthy, and Theodore Baas

Asymptotics of Visibility in Three-Dimensional Grid Worlds

Mentor: Dr. Zarathustra Brady

Project suggested by Dr. Zarathustra Brady

We focus on $\mathbb{N}^3$. Each unit cube whose vertices are lattice points is in one of two states: completely obstructed, or completely empty. A cube C will be considered visible if *any point* of the cube can be connected to the observer's cube by a straight line that does not pass through any obstructed cubes. We show that the lower bound of the maximum number of visible obstructions in an $N \times N \times N$ grid is $O\left(N^{\frac{8}{3}}\right)$. We first discuss the proof of the two dimensional analog of our problem, and some related material. We present a model that uses only parallel projections to the boundary squares of the grid, and prove that the maximum number of visible obstructions is bounded below by the claimed growth complexity. We then conclude by presenting our work on the upper bound of this model.

Session 2: Geometry and Topology

Ryan Kim

Cohomology Groups of the Dual Steenrod Algebra

Mentor: Sanath Devalapurkar

Project suggested by Prof. Haynes Miller

In homological algebra, the cohomology groups of a group action on a module are one of the most powerful structures available for understanding the action. We study the modulo 2 dual Steenrod algebra $\mathcal{A}_*$, a structure in homological algebra formed by taking the dual of the Steenrod algebra for $p = 2$. Milnor previously showed that the dual Steenrod algebra is a polynomial ring over $\mathbb{Z}/2$, and we investigate the cohomology groups of the canonical conjugation action χ of $\mathbb{Z}/2$ on $\mathcal{A}_*$. The first step of such a computation is computing $\mathcal{A}_*^\chi$, the fixed points subalgebra of $\mathcal{A}_*$ under χ. We introduce some important elements of $\mathcal{A}_*^\chi$, and we then study some asymptotics and bounds on the dimensions of $\mathcal{A}_*$ of $\mathcal{A}_*^\chi$.

Daniel Liu

New Fusion Products For the Affine Kac-Moody Algebra $\widehat{\mathfrak{sl}_2}$ *at Level* $\kappa = 1/2$.

Mentor: Dr. Claude Eicher

Project suggested by Dr. Claude Eicher

Fusion is a basic operation for representations of affine Kac-Moody algebras that resembles the tensor product. It can be motivated by two dimensional conformal field theory associated to these algebras, the so-called WZW models. In this project we restrict ourselves to the simplest case of the affine Kac-Moody algebra $\widehat{\mathfrak{sl}_2}$. At positive rational, but nonintegral, level κ fusion has been explored recently by Gaberdiel at $\kappa = -\frac{4}{3}$ and Ridout at $\kappa = -\frac{1}{2}$ using purely algebraic methods, the so-called algorithm of Gaberdiel-Kausch-Nahm, based on some natural assumptions. Among the fusion products indecomposable representations occur that are neither simple nor of highest weight. We compute such fusion products for the affine Kac-Moody algebra $\widehat{\mathfrak{sl}_2}$ at level $\kappa = \frac{1}{2}$ following the same techniques. We use MAGMA to compute the (relaxed) singular vectors, fusion to grade zero and to higher grades. Our ultimate goal is to show that fusion closes on a certain family of modules and to describe all fusion products.

Jiwon Choi

Gromov-Hausdorff Distance between Metric Spaces

Mentor: Ao Sun

Project suggested by Ao Sun

In this talk we study the Gromov-Hausdorff distance between two metric graphs. We compute the precise value of the Gromov-Hausdorff distance between two strings.

Sathwik Karnik

Bounds on the Maximal Cardinality of an Acute Set in a Hypercube

Mentor: Ao Sun

Project suggested by Prof. Larry Guth

The acute set problem asks the following question: what is the maximal cardinality of a d-dimensional set of points such that all angles formed between any three points are acute? In this project, we consider an analogous problem with the condition that the acute set is a subset of a d-dimensional unit hypercube. We provide an explicit construction and proof to show that a lower bound for the maximum cardinality of an acute set in $\{0,1\}^d$ is $2^{2^{\lfloor \log_3 d \rfloor}}$. Using a similar construction, we improve this lower bound to $2^{d/3}$. Through a consideration of points diagonally opposite a particular point on 2-faces, we improve the upper bound to $\left(1 + \frac{2}{d}\right) \cdot 2^{d-2}$. We then seek to generalize these findings and a combinatorial interpretation of the problem in $\{0,1\}^d$.

Session 3: Knot Theory

Max Guo

On Khovanov Homology, Bar-Natan's Perturbation, and Conway Mutation

Mentor: Dr. Jianfeng Lin

Project suggested by Dr. Jianfeng Lin

The Bar-Natan homology is a perturbation of the Khovanov homology of a knot. Previous work has shown that Khovanov homology remains unchanged under Conway mutation of the knot diagram, and we show this result holds true for the unreduced Bar-Natan homology over $\mathbb{Z}/2\mathbb{Z}$ as well. Our proof follows that of Lambert-Cole's work.

Aayush Karan

Mutation Invariance of the Szabó Spectral Sequence

Mentor: Dr. Jianfeng Lin

Project suggested by Dr. Jianfeng Lin

Homological algebra is an extremely useful tool in Knot Theory, allowing us to reinterpret geometry through the lens of algebra by assigning algebraic objects to knots. One particularly potent object is the Khovanov Homology, which is a knot invariant, or characteristic of a knot regardless of its continuous deformations. We can also consider deformations that break the knot's physical structure, a prime example being Conway Mutation. Surprisingly, the homology is also invariant under this mutation. An extension of the Khovanov Homology is Szabó's Spectral Sequence, which is a knot invariant as well. This presentation will provide a brief overview of the constructions required for these algebraic structures and pose the question of whether the Spectral Sequence is mutation invariant too.

Nithin Kavi and Wendy Wu

Width and Trunk of Satellite Knots

Mentor: Zhenkun Li

Project suggested by Zhenkun Li

We consider how the wrapping number of a satellite knot relates to its trunk number and its companion knot's trunk number. This relationship has been previously been observed with respect to winding number as opposed to wrapping number, and an inequality was determined. This inequality establishes a lower bound for the trunk number of a satellite knot in terms of the trunk number of the companion knot. Our results show that when replacing the winding number of the satellite knot by the wrapping number, we must include a constant factor of $\frac{1}{2}$ for the same inequality to hold true.

Sherry Lim and Mirilla Zhu

Permutations for Preschoolers: Applications of Group Theory to Childhood Games

Mentor: Margalit Glasgow

Expository Talk

This talk will explore the properties of the symmetric and alternating groups. We will then see how group theory can help us understand situations that arise in the real world by using the structure of the alternating group to determine the possible configurations of the Fifteen Puzzle.

Sekai Carr and Laura Clervil

Connect Spokes: An Original Combinatorial Game

Mentor: Marisa Gaetz

Expository Talk

We study an original combinatorial game called "Connect Spokes." Connect Spokes is an impartial, normal play game played by adding edges to a simple graph according to a set of rules. We present a winning strategy for Player 2 that is derived through an analysis of the terminal positions of the game, and how Player 2 can demand that the game lead to his/her terminal positions.

Elizabeth Euwart and Anna Rasmussen

Graphs and the Chromatic Polynomial

Mentor: Dhruv Ranganathan

Expository Talk

Our goal is to discuss the basic concepts behind graph theory and then to use these to examine the chromatic polynomial of a graph. We will begin by defining graphs and other important terminology, including the concept of coloring graphs, and then we will discuss the chromatic polynomial. Additionally, we will find the chromatic polynomials for some special categories of graphs, and introduce the concept of Read's Conjecture.

Carly Maggiolo Cabrera and Elisandra Fontes

Probability, Statistical Tests, and Fair Die

Mentor: Alexis Oriole

Expository Talk

We will be explaining the basics of probability and statistical tests. Probability measures the likelihood of an event happening. Statistical tests help evaluate data by comparing the results of an experiment to hypotheses. Goodness of Fit tests are a specific type of test that let you know whether your expectation of the probability of an event is accurate or not. There are 4 kinds of goodness of fit tests, but we focus on the Chi Square goodness of fit test. We used this test on four weird looking dice to determine if they were fair, and we found that all four dice were fair.

Rinni Bhansali

Modelling Epidemics on Polluted Networks and m-State Networks

Mentor: Prof. Laura Schaposnik, University of Illinois at Chicago

Project suggested by Prof. Laura Schaposnik

Bootstrap percolation describes an infection process across an nxn graph, where a site is infected once two of its neighbors are infected. In this project, we focus on two variations of standard bootstrap percolation. The first is polluted bootstrap percolation, where we account for the existence of vaccinations and seek to optimize methods of vaccine delivery. The second is m-state bootstrap percolation, where we categorize vertices into states (such as male and female), and define the update function as a site being infected once it has infected neighbors of each state. Here, we study the growth of infection along with time of percolation and probability of full percolation (the entire graph is infected).

Vincent Huang

Mathematical and Algorithmic Models of Refugee Crises

Mentor: Prof. James Unwin, University of Illinois at Chicago

Project suggested by Prof. James Unwin

The economic disparities between different parts of the world and spontaneous violent ethnic or political upheavals have made migration one of the most significant issues of the modern age. The primary cause of migration is the movement of individuals (in large number) each aiming to maximize their own safety and/or quality of life. Although the political and moral questions regarding migration are complicated, the mathematics of modeling the population flow is highly interesting. Moreover, given a well developed model which can predict the flow of migrants from some geographical region into neighboring districts, if one can reasonably predict the flux of people along particular routes and their end points, this could be invaluable for contingency planning such as asylum registration, enforcement, and humanitarian aid.

Kaiying Hou

Agent-Based Models for Conservation Equations

Mentor: Andrew Rzeznik

Project suggested by Andrew Rzeznik

In this research, we use agent-based models to solve conservation equations. A conservation equation is a partial differential equation that describes any conserved quantity by establishing a relationship between the density and the flux. It is used in areas such as traffic flow and fluid dynamics. Past research on numerically solving conservation equations mainly tackles the problem by establishing discrete cells in the space and approximating the densities in the cells. In this research, we use an agent-based model, in which we describe the solution through the movement of particles in the space. We propose an agent-based model for conservation equation in 1-D space, show that the agent-based solution converges to the actual solution in the scalar case, and demonstrate some preliminary result on the vector case.

Melinda Sun

Counting Bimonotone Subdivisions

Mentor: Dr. Elina Robeva

Project suggested by Dr. Elina Robeva

The number of bimonotone subdivisions compared to the total number of subdivisions of a point configuration provides insight into the correlation of random variables related to the configuration. We count both numbers for grid configurations in the plane, beginning with $2 \times n$ grids. We find a recursion and use it to prove the form of the expressions for the number of bimonotone and total subdivisions.

Haneul Shin

Bimonotone Subdivisions in High Dimensions

Mentor: Dr. Elina Robeva

Project suggested by Dr. Elina Robeva

A function is supermodular if for every u and v in $\mathbb{R}^d$,

$$f(u) + f(v) \leq f(\min(u, v)) + f(\max(u, v)).$$

Supermodular functions can be applied to statistics; given random variables $X_1, ..., X_n \in \mathbb{R}$, the distribution of the random vector $(X_1, \ldots, X_n)$ is given by a density function $p : \mathbb{R}^n \to \mathbb{R}$. If this density function $p(x) = \exp(f(x))$, where f is a supermodular function, then the random variables $X_1, \ldots, X_n$ are positively dependent. In order to characterize these supermodular functions, we study bimonotone subdivisions. It was previously shown that a tent function of a subdivision is supermodular if and only if the subdivision it induces is bimonotone. We use a computer program to determine when a certain subdivision is bimonotone to characterize the set of heights that induce a bimonotone subdivision.

Stanley Wang

Moduli Space of Planar Tropical Curves of Genus 1

Mentor: Yu Zhao

Project suggested by Dr. Dhruv Ranganathan

Tropical geometry is a relatively recent field created as a simplified model for certain problems in algebraic geometry. We will introduce the definition of certain objects in tropical geometry such as abstract and planar tropical curves and their properties such as combinatorial type, degree, and moduli space. Our research focuses on the moduli space of planar tropical curves with genus ≤ 1 and we prove that the moduli space of all planar tropical curves of degree 1 is connected.

Gopal Goel and Andrew Yao

Derivatives of the Gaussian Free Field via Random Matrix Theory

Mentor: Andrew Ahn

Project suggested by Prof. Vadim Gorin

This talk is in the area of Random Matrix Theory. The fundamental object of study is the Gaussian Orthogonal Ensemble (GOE), which is a symmetric random matrix whose upper triangular entries are independent and identically distributed centered Gaussians. The spectral statistics of high dimensional random matrices have shown deep connections with physics over the past century. It was shown by Borodin that the eigenvalues of the GOE converge to the Gaussian Free Field (GFF). We show that this limit is robust under differentiation, and we also talk about the combinatorics of unicyclic graphs that is used.

Session 7: Representation Theory and Number Theory

Yuting (Emma) Qin

The Elliptic Kashiwara-Vergne Lie Algebra

Mentor: Dr. Florian Naef

Project suggested by Dr. Florian Naef

In this talk, we present the graphical representations used to understand the elliptic Kashiwara-Vergne Lie algebra (krv). We start with the graphical representations of terms in the associative algebra, and then look at rooted Lie trees which represent terms in the lie algebra. From the trees, we explain the definition of the elliptic Kashiwara-Vergne Lie algebra. We finish with the current progress on elements of small degree in elliptic krv that we found, and future goals of the project.

Byung Yeon Rhee

Elliptic Curves, Factorization, and Cryptography

Mentor: Yongyi Chen

Expository Talk

In this talk, we discuss rational points on elliptic curves. An elliptic curve is the set of solutions to a cubic equation in the projective plane, along with a distinguished point at infinity. We are able to create an Abelian group out of the rational points of such curves. We then give applications of this theory to an integer factorization algorithm before ending with another discussion regarding another application of the elliptic curve in modern cryptography.

Merrick Cai

The Hilbert Series of the Irreducible Representation of the Rational Cherednik Algebra of Type A_{n-1} in Characteristic p

Mentor: Daniil Kalinov

Project suggested by Prof. Pavel Etingof

We examine the polynomial representation of the rational Cherednik algebra of type A_{n-1} of generic parameter in positive characteristic p where p does not divide n. We aim to find the Hilbert series of an irreducible representation $L_{t,c}(\tau)$. We look to find singular polynomials, or polynomials which are in the simultaneous kernel of the Dunkl operators. We prove the existence of specific types of singular polynomials for cases $p = 2$ and n odd, and $p|n-1$ for p odd.

Archer Wang

Hilbert Series of Quasi-Invariant Polynomials

Mentor: Dr. Xiaomeng Xu

Project suggested by Prof. Pavel Etingof

The space of quasiinvariant polynomials of order m generalizes that of symmetric polynomials: under the action of the symmetric group, the polynomials remain invariant to a certain order. Their Hilbert series were studied by Felder and Veselov in fields of characteristic 0. In this paper, we investigate the Hilbert series of quasiinvariant polynomials that are divisible by a generic homogeneous polynomial. In addition, we continue the work on previous results regarding their Hilbert series in fields of prime characteristic.

SESSION 8: COMBINATORICS

Vinjai Vale

A New Paradigm for Computer Vision Based on Compositional Representation

Mentor: Kevin Ellis

Project suggested by Vinjai Vale

Deep convolutional neural networks — the state-of-the-art technique in artificial intelligence for computer vision — achieve notable success rates at simple classification tasks, but are fundamentally lacking when it comes to representation. These neural networks encode fuzzy textural patterns into vast matrices of numbers which lack the semantically structured nature of human representations (e.g. "a table is a flat horizontal surface supported by an arrangement of identical legs"). This paper takes multiple important steps towards filling in these gaps. I first propose a series of tractable milestone problems set in the abstract two-dimensional ShapeWorld, thus isolating the challenge of object compositionality. Then I demonstrate the effectiveness of a new compositional representation approach based on identifying structure among the primitive elements comprising an image and representing this structure through an augmented primitive element tree and coincidence list. My approach outperforms Googles state-of-the-art Inception-v3 Convolutional Neural Network in accuracy, speed, and structural representation in my object representation milestone tasks. Finally, I present a mathematical framework for a probabilistic programming approach that can learn highly structured generative stochastic representations of compositional objects from just a handful of examples. This work is foundational for the future of general computer vision, and its applications are wide-reaching, ranging from autonomous vehicles to intelligent robotics to augmented and virtual reality.

Ben Chen, Richard Chen, Joshua Guo, Shane Lee, Neil Malur, Nastia Polina, Poonam Sahoo, Anuj Sakarda, Nathan Sheffield, and Armaan Tipirneni (PRIMES STEP Senior students)

On Base 3/2 and Its Sequences

Mentor: Dr. Tanya Khovanova

Project suggested by Dr. Tanya Khovanova

We will talk about how base 3/2 behaves, and how base 3/2 can create some cool sequences from well-known ones. We will discuss a fundamental tree related to this base and its connections to patterns in integers written in base 3/2. We also cover Fibs sequences and show their eventual behaviors. We will tell you about Pinocchio and Oihcconip sequences.

Matvey Borodin, Hannah Han, Kaylee Ji, Alexander Peng, David Sun, Isabel Tu, Jason Yang, William Yang, Kevin Zhang, and Kevin Zhao
(PRIMES STEP Junior students)

Chips Go BOOM BOOM!!!

Mentor: Dr. Tanya Khovanova

Project suggested by Dr. Tanya Khovanova and Prof. James Propp, UMass Lowell

The use of explosives has been utilized for centuries throughout history for horrendous purposes. However, we will now properly restore their righteous redemption of explosions by using them in fractional bases and chip firing.

Session 9: Combinatorics

Vincent Bian

Packing Anchored Rectangles

Mentor: Dr. Tanya Khovanova

Project suggested by Prof. Yufei Zhao

A set of points, including the origin, is chosen in $[0, 1]^2$. An anchored rectangle packing is a set of non-overlapping rectangles in the unit square, such that the lower left corner of each rectangle is one of the chosen points, and no rectangle contains a chosen point on its interior. We try to solve a 50 year old conjecture that any set of points has an anchored rectangle packing with total area at least $\frac{1}{2}$, and prove it for special configurations of points.

Eric Zhang

On Quasirandom Permutations

Mentor: Dr. Tanya Khovanova

Project suggested by Prof. Yufei Zhao

What does it mean for a permutation to be quasirandom? One of the ways of determining this is by looking at patterns of smaller permutations within sequences. We will discuss the challenges of building a sequence of quasirandom permutations and find a weaker sufficient condition for quasirandomness based on the densities of a subset of length-4 pattern permutations.

Wayne Zhao

Counting Sudoku Variants

Mentor: Dr. Tanya Khovanova

Project suggested by Dr. Tanya Khovanova

We discuss a new Sudoku variant called Sudo-Kurve and different mathematical aspects of it.

William Fisher

Polynomial Wolff Axioms and Kakeya-Type Estimates for Bent Tubes

Mentor: Robert Burklund

Project suggested by Prof. Larry Guth

We consider the recent work due to Guth and Zahl involving the polynomial Wolff axioms and investigate its applications to bounds on curved tube. We show that their work automatically extends to algebraic curves with bounded curvature, and consider its ability to control sets of C^k tubes with bounded curvature.

Alan Yan

Asymptotic Counting in Dynamical Systems

Mentor: Prof. Sergiy Merenkov, CCNY – CUNY

Project suggested by Prof. Sergiy Merenkov

We consider several geometric objects with certain measurable properties. We define various counting functions on these geometric objects and explore the asymptotics of these functions. We conjecture that these functions grow like power functions with the exponent as the dimension of the residual set and we provide an algorithm to numerically verify this conjecture. The main objects that we will be examining will be Fatou components of the quadratic family and limit sets of Schottky groups.

Andy Xu

Approximating the Hurwitz Zeta Function

Mentor: Hyun Jong Kim

Project suggested by Dr. Stefan Wehmeier, Mathworks

In this project, we aim to approximate the Hurwitz Zeta function over all inputs. This is necessary because the naive implementation fails for certain input near critical values for s and for a. Other series representations of the Hurwitz Zeta function converge rapidly but do not handle complex values of s and/or a. We also consider existing forms for the Hurwitz Zeta function, including one given by Bailey and Borwein, and evaluate their overall performance.

Aurash Vatan

Finding Generators of Jacobian groups of Finite Graphs

Mentor: Dr. Xiaomeng Xu

Project suggested by Dr. Dhruv Ranganathan

Each finite connected graph without loops has an associated canonical Abelian group called the Jacobian (analogous to the Jacobian groups of Riemann surfaces). The elements of this group are "divisors" of the graph (assignments of integers to the nodes) modded-out by an equivalence relation called chip-firing. Previous work by Brandfonbrener et. al. shows how to determine the generator of most cyclic Jacobians. We explore a new procedure for producing generating sets of Jacobians.

Daniel Zhu

On the Okounkov-Olshanski Formula for the Number of Tableaux of Skew Shapes

Mentor: Prof. Alejandro Morales, UMass Amherst

Project suggested by Prof. Alejandro Morales

Standard Young tableaux of ordinary and skew shapes are fundamental objects in algebraic combinatorics and have applications to a variety of other fields. The hook-length formula of Frame, Robinson, and Thrall, proven in 1954, gives a product formula for the number of standard tableaux of ordinary shapes. However, there is no general product formula for skew shapes. In 1996, Okounkov and Olshanski found a nonnegative formula for the number of standard Young tableaux of a skew shape. We prove properties of this formula, including a determinantal formula for the number of nonzero terms and a q-analogue extending the Okounkov-Olshanski formula to reverse plane partitions. The latter result complements work by Chen and Stanley for semistandard tableaux of skew shape.

Allen Wang

Permutations with Up-Down Signatures of Nonnegative Partial Sums

Mentor: Guangyi Yue

Project suggested by Prof. Richard Stanley

An up-down signature, or Niven signature, of a permutation $w \in \mathfrak{S}_n$ is a sequence of $+1$ (resp. -1) that correspond to ascents (resp. descents) in w. Many, including Shevelev and Szpiro, have derived formulae for the number of permutations of a fixed signature and the expectation of their elements. We extend their previous work by investigating permutations whose up-down signatures have nonnegative partial sums. In 2006, Callan conjectured that the enumeration of such permutations in $\mathfrak{S}_n$ matches sequence A000246 of the OEIS. These numbers satisfy the recursive relationship

$$f(1) = 1,\ f(2n) = (2n-1)f(2n-1),\ \text{and}\ f(2n+1) = (2n+1)f(2n),$$

which we verify.

Vivek Bhupatiraju

accAAD: Efficient Append-Only Authenticated Dictionary for Transparency Logs

Mentor: Alin Tomescu

Project suggested by Alin Tomescu

Clients using public logs should be able to securely verify that data in the log has not been maliciously changed or removed by the logs server. Unfortunately, verification in these so-called transparency logs is generally computation efficient for the server but bandwidth inefficient for the clients. As client-side bandwidth is far more expensive than server-side computation, we attempted to design a bandwidth efficient transparency log. In this talk, we present our novel cryptographic primitive called an append-only authenticated dictionary (AAD), which details what proofs the server must provide for the client. We then briefly discuss accAAD, an efficient AAD based on bilinear accumulators. accAAD presents significant improvements over existing transparency logs, indicating it can be implemented in important logs like secure public-key directories and certificate authorities.

Theodor Lukin Yelin

Designing Private Forums Using Differential Privacy

Mentor: Albert Kwon

Project suggested by Albert Kwon

In the modern day, forums such Reddit and Facebook have become integral part of our daily lives. Much of their success has come from collecting user's votes and views, which allow them to determine the best content to present to others. However, it is often possible to profile people and track their interests based of what they are viewing and voting on.

In our work, we investigate the use of a concept known as Differential Privacy in order to obscure users individual usage patterns, while still allowing the forums to collect necessary information about general behavior. By adding random noise to the data collection procedure, we can provide accurate estimates on the true aggregate statistics of the users' behaviors while hiding the individual actions of each user.

Sanjit Bhat and David Lu

Var-CNN and DynaFlow: Improved Attacks and Defenses for Website Fingerprinting

Mentor: Albert Kwon

Project suggested by Albert Kwon

In recent years, there have been many works that use website fingerprinting techniques to enable a local adversary to determine which website a Tor user is visiting. However, most of these works rely on manually extracted features, and thus are fragile: a small change in the protocol or a simple defense often renders these attacks useless. In this work, we leverage deep learning techniques to create a more robust attack that does not require any manually extracted features. Specifically, we propose Var-CNN, an attack that uses model variations on convolutional neural networks with both the packet sequence and packet timing data. In open-world settings, Var-CNN attains higher true positive rate and lower false positive rate than any prior work at 90.9% and 0.3%, respectively. Moreover, these improvements are observed even with low amounts of training data, where deep learning techniques often suffer.

Given the severity of our attacks, we also introduce a new countermeasure, DynaFlow, based on dynamically adjusting flows to protect against website fingerprinting attacks. DynaFlow provides a similar level of security as current state-of-the-art and defeats all attacks, including our own, while being over 40% more efficient than existing defenses. Moreover, unlike many prior defenses, DynaFlow can protect dynamically generated websites as well.

Harshal Sheth and Andrew Sun

Extending Distributed-Systems Tracing Vertically into the Linux Kernel

Mentor: Dr. Raja Sambasivan, Boston University

Project suggested by Dr. Raja Sambasivan

Modern applications are often architected as a sprawling fleet of microservices. While this does have benefits, it also makes it incredibly difficult for developers to diagnose issues with their applications. Many tools exist to trace such distributed systems, but they miss an important part of application performance: the kernel. We present Skua, a modified suite of tracing utilities that gains insight into both application- and kernel-level behavior. Logging information produced by LTTng is augmented with tracing context information and integrated into the existing distributed tracing framework provided by Jaeger. In addition to demonstrating correctness and attempting to integrate it into nginx, Skua was found to cause a 14.9% decrease in average throughput in a small Web server, which is acceptable for many applications.

Session 13: Medical Informatics

Makiah Bennett

Lexical AI for Patient-Centered Clinical Diagnoses

Mentor: Prof. Gil Alterovitz

Project suggested by Prof. Gil Alterovitz

Modern medical applications are limited by their inability to truly interact with a user, relying heavily on human-created text selections to provide input, resulting in the necessity for a human diagnostician regardless of the ailment. Yet humans are, by definition, human, and modern diagnosticians are required to focus their minds on a plethora of patients over finite time spans, while yet other patients lack the time and/or funds to see their doctor as regularly as they should. We propose the development of a lexical neural network which is capable of independently analyzing and interpreting patient data, retraining itself on patient feedback, and incorporating extensive data into its analyses. The service will integrate extensively with the new Sync4Science initiative as well as previous work with SMART on FHIR to enable analysis and cross-reference of patient information. Simultaneously, we plan to incorporate libraries such as PyTorch and Word2Vec for interpretation of patient symptoms and patient tone, which use deep learning and multidimensional vector spaces for complex yet effective interpretation. Finally, we hope to see this service employed on smart home speakers and mobile devices, which will improve our ability to re-train models and provide users the opportunity to provide feedback on their medications unobtrusively.

Andrew Gritsevskiy

Capsule Networks for Low-Data Transfer Learning

Mentors: Prof. Gil Alterovitz and Maxim Korablyov

Project suggested by Prof. Gil Alterovitz and Maxim Korablyov

We propose a capsule network-based architecture for generalizing learning to new data with few examples. Using both generative and non-generative capsule networks with intermediate routing, we are able to generalize to new information over 25 times faster than a similar convolutional neural network. We train the networks on the multiMNIST dataset lacking one digit. After the networks reach their maximum accuracy, we inject 1-100 examples of the missing digit into the training set, and measure the number of batches needed to return to a comparable level of accuracy. We then discuss the improvement in low-data transfer learning that capsule networks bring, and propose future directions for capsule research.

Kevin Hu

Discovery of Exon Splicing Relationships Across Cancer Cell Lines

Mentor: Dr. Mahmoud Ghandi, Broad Institute

Project suggested by Dr. Mahmoud Ghandi

With the emergence of high-throughput sequencing techniques, several efforts have been made to compile datasets to annotate the various molecular characteristics of human cancer cell lines. In this study, we examine the roles of alternative RNA splicing in the Cancer Cell Line Encyclopedia (CCLE) against other features such as shRNA-knockdown gene dependencies, copy number variations, and gene expression. We describe our efforts to examine the impact of exon inclusion levels on dependencies observed at the shRNA/sgRNA level in the Achilles and Project DRIVE datasets. By incorporating RNA splicing levels, we hope to improve the characterization of sensitive oncogenes by accounting for an additional dimension of genetic regulation.

Session 14: Math Reading Groups

Nhat Pham and Anmol Sakarda

Concrete Mathematics: Exploring Summations

Mentor: Zhulin Li

Expository Talk

We explore various methodologies to calculate sums. We begin by looking at simple methods: converting sums into recurrences. We proceed to discuss common manipulations, such as using the repertoire method or perturbation. Finally, we conclude by extrapolating sums to both finite and infinite calculus mainly by building integrals.

Dylan Pentland

The j-Invariant of an Elliptic Curve

Mentor: Chun Hong Lo

Expository Talk

The j-invariant $j(E)$ of an elliptic curve $E/\mathbb{Q}$ determines the isomorphism class of the curve over $\mathbb{C}$, or more accurately the isomorphism class over $\overline{\mathbb{Q}}$. For the (nonsingular) elliptic curve $y^2 = x^3 + ax + b$, this is given by $j(E) = 1728\frac{4a^3}{4a^3+27b^2}$.

We will motivate the definition of the j-invariant by using the correspondence between elliptic curves and lattices $\Lambda \subset \mathbb{C}$ and showing how the j function appears naturally in this new context.

Session 15: PRIMES Circle

Aneesha Manne and Lara Zeng

Generating Functions in Combinatorics

Mentor: Uma Roy

Expository Talk

Generating functions are a powerful tool in combinatorics and can be used for solving a wide variety of tasks, including getting closed forms for recurrences, deriving asymptotic formulae and studying other properties of sequences. We introduce basic notions for generating functions and exponential generating functions and present some of their applications to solving the aforementioned problems.

Iris Yang and Victoria Zhang

Gaussian Integers and Their Relationship to Ordinary Integers

Mentor: Matthew Weiss

Expository Talk

This talk will explore Gaussian integers and make analogies to the ordinary integers. With the ultimate intent of proving unique factorization for Gaussian integers, we discuss units, primes, and greatest common divisors, among others, as well as draw parallels to the ordinary integers.

Sam Costa and Elin Gu

Inverse Knots and Amphicheirality

Mentor: Joseph Zurier

Expository Talk

This talk will focus on knot comparison, and how tricolorability and polynomials can be useful in that respect. We will show how using polynomials can either distinguish inverse knots or confirm that a knot possesses a property called amphicheirality, which we will explain.

We will prove the unknot is distinct from the trefoil with tricolorability, and prove the trefoil is distinct from its inverse using a polynomial which we will derive, to conclude that the trefoil is not amphicheiral.