An Algorithmic and Computational Approach to Optimizing Gerrymandering

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Introduction

What and Why: Voting Districts in Democracy

- Determine elected representatives
- Equal population
- Determined periodically by
  - State legislature
  - Independent commission
- Analogous to electoral college

Maryland’s 3rd District
(Source: Wikipedia)
**Importance**

**Misrepresentation:** districting can lead to unbalanced districts, making certain voters ineffective.

**Gerrymandering** exploits misrepresentation for political gain.

“We have to end the practice of drawing our congressional districts so that politicians can pick their voters, and not the other way around.”

– Barack Obama
Impacts of Gerrymandering

Kyle Gatesman

Optimizing Gerrymandering
Friedman-Holden Approach

We considered a specific approach to optimal gerrymandering:

The Friedman-Holden districting approach is based on

- Extremity
- Continuous distribution
- Shock factor model
- Population continuity
- Geography not accounted for

We first aim to study the geographic distribution of districts that arise.

Friedman, Holden, Am Econ Rev. (2008), 98:1, 113-144
Implementation: Voter Distribution

To study the districts arising from Friedman-Holden approach we take

- Population on a lattice
- Lattice units will be associated to either Proponent or Opponent.
- Distribution from probabilistic walker method
- Randomized, but mimics a 2D Gaussian distribution

We take an $11 \times 11$ lattice and all voters “walk” from the center point. This gives an overall normal distribution, with small fluctuations:

```
  1  4  7  4 14 15 13  8  6  3  2
 4 11 15 21 26 30 31 19 12  8  3
 4 13 22 38 41 44 33 42 26  6  5
 7 22 28 61 70 87 60 63 42 23  7
 7 17 42 63 126 108 138 68 52 40 21
 7 27 57 87 99 149 95 90 40 38 13
 8 16 34 76 124 110 105 69 57 27 11
12 26 37 69 79 86 75 61 34 19  6
 6 10 16 38 49 47 44 34 25 11  8
 4 10 12 31 34 36 19 28 19 11  3
 2  2  8 10 10 11 10 14  6  2  4
```
Implementation: Partisanship

We assign some population unit $P$ to be a source of partisan bias $E_P$.

**Contribution to extremity** at point $Q$, a distance $d(P, Q)$ away, is

$$\Delta E(Q) = E_P / d(P, Q).$$

The voter extremity at point $Q$ is a sum over all sources

$$E_{\text{net}}(Q) = \sum_{P \in S} \frac{E_P}{\max[1, d(P, Q)]}$$

This draws on an analogy to electrostatic potential.

Take an idealized model with symmetric and proximal source points:
Friedman-Holden Approach to Districting

Combining the population and parity distributions, one obtains the aggregate vote distribution, example (with net-vote $+2.39$):

```
-0.03  -0.12  -0.21  -0.1  -0.19  0.0  0.18  0.2  0.18  0.09  0.06
-0.15  -0.48  -0.71  -0.90  -0.68  0.0  0.81  0.82  0.56  0.35  0.11
-0.20  -0.83  -1.69  -3.08  -2.29  0.0  1.85  3.4  2.00  0.38  0.26
-0.46  -2.02  -3.64  -10.37 -10.22  0.0  8.76  10.71  5.46  2.12  0.46
-0.55  -2.04  -8.61  -24.63 -69.68  0.0  76.31  26.59  10.66  4.8  1.64
-0.58  -3.59  -14.25 -58.03  -49.5  0.0  47.5  60.03  10.0  5.05  1.08
-0.62  -1.92  -6.97  -29.72 -68.57  0.0  58.07  26.98  11.69  3.24  0.86
-0.78  -2.39  -4.81  -11.73 -11.53  0.0  10.95  10.37  4.42  1.75  0.39
-0.31  -0.64  -1.23  -3.08  -2.74  0.0  2.464  2.75  1.925  0.70  0.41
-0.15  -0.44  -0.56  -1.33  -0.88  0.0  0.494  1.2  0.893  0.48  0.11
-0.06  -0.06  -0.24  -0.25  -0.14  0.0  0.14  0.35  0.18  0.06  0.11
```

We wish to district the above such that the proponent party wins.

Our algorithmic implementation of Friedman-Holden involves:

- A preset population benchmark per district
- Chunking of territorial units
- Fine-tuning
- Recursion on subsequent, moderate districts
Program Results: Districting

Visualization of the **five districts** determined by our algorithm
Limitations

As anticipated, application of the unmodified Friedman-Holden approach does not satisfy geographic restrictions:

- Continuity (legally required)
- Compactness
- Convexity

As example, see District 4 (right).

We are refining our algorithm to better adapt to a realistic setting.

We hypothesize that the efficacy of gerrymandering becomes more limited when more constraints need to be satisfied.
Summary and Future Directions

We have studied the Friedman and Holden approach to gerrymandering.

Our lattice study shows Friedman-Holden leads to non-continuous districts.

We are working to construct an algorithm incorporating restrictions on districts from:

- Continuity
- Compactness
- Convexity

We aim to show these constraints make gerrymandering more difficult.

Studying gerrymandering methods will aid in detecting and inhibiting it.
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