Investigating Mixed Memory-Reinforcement Models for Random Walks

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Sixth Annual MIT-PRIMES Conference
May 21, 2016
Random Walks in Biology

Definition

A random walk is a path that consists of a series of random steps.

Examples

Path of a molecule in a gas
Motion of a slime mold towards food
Movement of ants between food source and anthill

Not necessarily purely random
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Memory and Reinforcement

Memory: higher probability of moving along same direction of motion
Favors boundaries of environment

Reinforcement: higher probability of moving along previous paths taken by other particles
Ants following trails of chemical pheromone
Causes slower spread of particles away from starting location
Memory and Reinforcement

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Goals

Memory and reinforcement have been studied separately. Build a model in which memory and reinforcement are both factors. More realistic biologically. Possible optimum memory-reinforcement mix for least travel time. Reproduce and explain phenomena such as death spiral.
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Investigating Mixed Memory-Reinforcement Models
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  - Possible optimum memory-reinforcement mix for least travel time
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Memory Models

- Memory involves angles $\theta$ of deflection
- Memory parameter $m$

**Figure:** General intersection in a graph
Memory Models

- Simple model: Rectangular Grid
Memory Models

\[ \frac{1}{4}(3m + 1) \]

\[ \frac{1}{4}(1 - m) \]

\[ n_i \]

\[ \frac{1}{4}(1 - m) \]
Memory Models

- General graph

Figure: General intersection
Memory Models

- Assign $U$ and $L$ weights to forward and backward directions.

Figure: General intersection

\[ W(\theta) = (U - L) f(\theta) + L \]
Memory Models

- Assign $U$ and $L$ weights to forward and backward directions

![Diagram showing $L$ and $U$ with angle $\theta$](image)

**Figure:** General intersection

$$W(\theta) = (U - L)f(\theta) + L$$
Memory Models

\[ f(\theta) = e^{\frac{U-1}{2-U}(\pi-\theta)} - 1 \]

Figure: Graph of \( f(\theta) \) vs. \( \theta \)

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Memory Models

\[ f(\theta) = \frac{e^{\frac{U-1}{2-U}(\pi - \theta)}}{e^{\frac{U-1}{2-U}\pi} - 1} \]

Figure: Graph of \( f(\theta) \) vs. \( \theta \).
Memory Models

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Reinforced Random Walks

Probability depends on pheromone concentration ($d_{ij}$) and edge length ($l_{ij}$).
Reinforced Random Walks

- Probability depends on pheromone concentration ($d_{ij}$) and edge length ($l_{ij}$)

**Figure**: General edge $E_{ij}$
Reinforced Random Walks

- Mean flow rate ($\bar{I}_{ij}$) equation based on edge weights (Ma Q, et. al.):

$$
\bar{I}_{ij} = \left( \frac{N_i}{\sum_{e \in E_i} \frac{d_e}{l_e}} - \frac{N_j}{\sum_{e \in E_j} \frac{d_e}{l_e}} \right) \left( \frac{d_{ij}}{l_{ij}} \right)
$$

- $N_i$ is the number of particles at node $n_i$
- $E_i$ is the set of all edges around node $n_i$
- $d_e$ is the pheromone concentration on an edge $e$
- $l_e$ is the length of an edge $e$
Mixed Model

Weighted average of pure memory and pure reinforcement probabilities

Memory weight is $m$

Reinforcement weight is $1 - m$
Mixed Model

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Results

Pure memory on 7 x 7 grid: memory=1.0, reinforcement=1, evaporation=0.01, reflection=0.1, times=1000

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Investigating Mixed Memory-Reinforcement Models
Results

- **Pure memory** on 7 x 7 grid: memory=1.0, reinforcement=1, evaporation=0.01, reflection=0.1, times=1000
Results

- **Pure reinforcement** on $7 \times 7$ grid: memory=0, reinforcement=1, evaporation=0.01, reflection=0.1, times=1000
Results: Mixed Memory Reinforcement Model

(a) $m = 0$

(b) $m = 0.1$

(c) $m = 0.3$

(d) $m = 0.4$
Results: Mixed Memory Reinforcement Model

(a) $m = 0.5$
(b) $m = 0.6$
(c) $m = 0.7$
(d) $m = 1$

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Investigating Mixed Memory-Reinforcement Models
Shortest Arrival Time

Fractional Arrival Time (0.02) vs. Memory

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Generalized Graph Model

- Early results
Future Work

Further simulations on generalized graph model
Finding optimum memory-reinforcement mixes on general graphs
Refining optimum mix analysis on rectangular grids
Comparing and validating key simulation results with experimental results
Constructing a continuous-space, continuous-time model with partial differential equations rather than discrete time-steps
Reproducing the death spiral
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Acknowledgements

Thanks to

- My mentor Andrew Rzeznik
- My parents
- and the MIT-PRIMES program.