Modeling of Disease Spreading on Trees

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1 Introduction and Definitions

2 The Problem

3 Results and Continuation

4 References
Motivation:

Diseases
Spread of "information" (rumors, etc.)
→ the motivation for the question
Motivation

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- Diseases
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→ the motivation for the question
Graphs and Trees

Definition

**Graph**: A simple graph \((V, E)\) consists of a set representing vertices, \(V\), and a set of unordered pairs of elements of \(V\) representing edges, \(E\).
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**Tree**: A tree is a graph that is connected and has no loops.
Graphs and Trees

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**Graphs and Trees**

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**Lemma**

**Tree**: A tree is a graph with $|V| = |E| + 1$ and no loops.
Graphs and Trees

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More Definitions

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**Binary Tree**: A binary tree is a tree such that for each vertex $V$, there are at most 2 children.
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**Perfect Binary Tree**: A perfect binary tree is a binary tree with $2^N - 1$ vertices such that the last level is completely full.
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**Binary Tree**: A binary tree is a tree such that for each vertex $V$, there are at most 2 children.

Definition

**Perfect Binary Tree**: A perfect binary tree is a binary tree with $2^N - 1$ vertices such that the last level is completely full.

Note that such a tree is unique, not including labeling or directed edges.
Perfect Binary Tree
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Depiction of the Problem

- Given any tree, nodes $\equiv$ people
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- Nodes are represented with a Boolean variable for infection
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Depiction of the Problem

- Given any tree, nodes = people
- Nodes are represented with a Boolean variable for infection
- Infection can jump from any initial node to any other node in the sub-tree that it is the root of
  - Each node has a chance to be infected at any given point
- Infection rate decreases with distance from node
An Example

[Diagram of a tree structure with a green root node and multiple blue leaf nodes]

Modeling of Disease Spreading on Trees

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An Example
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Questions

1. How long will it take to reach layer \( n \) or below?
Questions

1. How long will it take to reach layer $n$ or below?
2. How many infected nodes are there when this occurs?
Solving the Problem(s)

- Program was devised to simulate the binary tree and infection process

\[ \text{\(k\) is the difference in layers between the infecting/infected nodes} \]
Solving the Problem(s)

- Program was devised to simulate the binary tree and infection process
  - Used MATLAB

\[
\alpha \text{ is a predetermined constant such that } \alpha > 1, \quad k = \text{the difference in layers between the infecting/infected nodes}
\]
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- Modeled the infection time between two points with exponential distributions
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  \[ 2^{-k} \left( k^{1-\alpha} - (k + 1)^{1-\alpha} \right) \]

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- Exponential distributions had rates of $2^{-k}(k^{1-\alpha} - (k + 1)^{1-\alpha})$
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  - $k$ is the difference in layers between the infecting/infected nodes
Solving the Problem(s)

There are 3 important properties of an exponential distribution that we can use
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**Property**

*Memoryless*: If $X = \exp(r)$, then $P(X > x + y | X > y) = P(X > x)$. 
Solving the Problem(s)

There are 3 important properties of an exponential distribution that we can use

Property

Memoryless: If \( X = \exp(r) \), then \( P(X > x + y | X > y) = P(X > x) \).

Property

Minimum: If \( X_1, X_2, \ldots, X_n \) are all exponential with rates \( r_1, r_2, \ldots, r_n \), then \( \min(X_1, X_2, \ldots, X_n) = \exp(r_1 + r_2 + \cdots + r_n) \).
Solving the Problem(s)

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**Property**

Memoryless: If $X = \exp(r)$, then $P(X > x + y | X > y) = P(X > x)$.

**Property**

Minimum: If $X_1, X_2, \ldots, X_n$, are all exponential with rates $r_1, r_2, \ldots, r_n$, then $\min(X_1, X_2, \ldots, X_n) = \exp(r_1 + r_2 + \cdots + r_n)$.

**Property**

Probability: If $X_1, X_2, \ldots, X_n$, are all exponential with rates $r_1, r_2, \ldots, r_n$, then the probability that $X_i$ is the minimum of $X_1, X_2, \ldots, X_n$ is

$$\frac{r_i}{r_1 + r_2 + \cdots + r_n}.$$
Properties can shorten simulation time

1. Pick a point that will infect, probability property
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Properties can shorten simulation time

1. Pick a point that will infect, probability property
2. Pick point to be infected
3. Time generated from $\exp(\text{sum of all infected node rates})$, minimum property
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The results seem to model a curve that is slightly skewed right
Results
Conjecture

If $1 < \alpha < 2$, prediction is polynomial in $n$ with degree $\alpha - 1$
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Conjecture

If $\alpha \geq 2$, prediction is linear in $n$
Future Goals

Expand to different types of trees
Acknowledgments

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