Cayley’s Formula

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<table>
<thead>
<tr>
<th>$n$</th>
<th>$A_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
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<tr>
<td>2</td>
<td>1</td>
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<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
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</tbody>
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Cayley’s Formula:

$A_n = n^{n-2}$
A rooted forest, viewed as a directed graph
- For each component, one vertex is called a root.
- Every edge is directed away from the root.
If, in $F'$, an edge starts at vertex $x$ and ends at vertex $y$, there also is an edge from $x$ to $y$ in $F$. 

$F$ contains $F'$.
A refining sequence $(F_1, ..., F_n)$
- Each forest $F_i$ contains $F_{i+1}$.
- Each forest $F_i$ has exactly $i$ components.
N: #rooted trees on n vertices
N*: #refinig sequences \((F_1,\ldots,F_n)\)
\[ N^* = N(n-1)! \]
N: #rooted trees on n vertices
N*: #refining sequences $(F_1, \ldots, F_n)$

\[ N^* = n(n-1) \times n(n-2) \ldots n \times 1 \]
\[ N^* = n^{n-1} (n-1)! \]
\[ N^* = N(n-1)! \]
\[ N = n^{n-1} \]
\[ N = A_n \times n \]
\[ A_n = n^{n-2} \]
A_{5}
$A_5 = 5^3 = 125$