STAGE seminar, fall 2014

Website: `http://math.mit.edu/nt/index_stage`

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Time: M 11-12 in 66-160.

Brief overview The seminar this fall will deal with periods and the fundamental group of the projective line minus three points. The three main players will be multiple zeta values, mixed Tate motives over $\mathbb{Z}$, and $\pi_1(\mathbb{P}^1 \setminus \{0, 1, \infty\})$. Of course, a large part of the seminar will be devoted to understanding what these objects are and how they are related.

The end goal of the seminar will be to prove a result of Francis Brown which can be phrased in two ways:

- Explicit description of certain generators for values of (generalizations of) Riemann's zeta function at integers;
- The motivic fundamental group of $\mathbb{P}^1 \setminus \{0, 1, \infty\}$ generates the category of mixed Tate motives over $\mathbb{Z}$.

Along the way, we will deal with Belyi's theorems, periods of algebraic varieties, basics on motives and fundamental groups.

References Brown's paper is

- F. Brown, *Mixed Tate motives over* $\mathrm{Spec}(\mathbb{Z})$, Ann. Math. **175**, n.1.

The results are surveyed in the following papers, which are great but still assume a lot of background:

- F. Brown, *Motivic periods and the projective line minus three points*, proceedings ICM 2014, arXiv e-print.
- P. Deligne, *Multizêtas, d'après Francis Brown*, Séminaire Bourbaki 1048, `http://www.bourbaki.ens.fr/TEXTES/1048.pdf`.

Belyi's theorem is proved in

- J.-P. Serre, *Lectures on the Mordell-Weil theorem.*

Basic references on the whole theory include

- P. Deligne, *Le groupe fondamental de la droite projective moins trois points*, available on Deligne's webpage (Deligne's fundamental paper)
- Y. Ihara, *Braids, Galois groups, and some arithmetic functions*, ICM proceedings 1990 (for an overview of the work following Deligne's paper above).
- P. Deligne, A. Goncharov, *Groupes fondamentaux motiviques de Tate mite*, A.S.E.N.S 2005 (quite difficult, we will use the first two paragraphs).

Lecture notes on the topic can be found at

- `http://math.arizona.edu/~swc/aws/2005/notes.html`

The references above will certainly need to be completed by more basic reading.