

18.336 Homework 4 :: Spring 2010 :: Due May 4

1. (100 pts) Consider the inviscid Burgers equation

$$u_t + u u_x = 0, \quad x \in [0, 1],$$

with initial condition $u(x, 0) = 3/2 + \sin(2\pi x)$, and periodic boundary condition. Implement the Euler explicit / upwind finite difference method, solve the equation to some large time after a shock forms, and produce a picture of the numerical solution. Discuss accuracy, i.e. (i) produce a plot of error vs. grid size h , and (ii) comment qualitatively on position and width of the numerical shock, and numerical diffusion or dispersion. Explain how your findings relate to the fact that the numerical scheme is not conservative.

Repeat the exercise with the Godunov finite volume method with piecewise constant reconstruction. Briefly describe the shock wave and rarefaction wave solutions on which you base your scheme.

If you typeset your report, make sure that it contains: at most one page of text, four figures, and an appendix with the code. If you handwrite your report, make sure that the amount of text you write is no more than if you had followed the instruction above.

[Hint: if you would like to have access to the explicit solution, look up the Cole-Hopf transformation.]