

# 18.085 Computational Science and Engineering

## Problem Set 1

Due in-class on 19th Feb 2015

Clarification required? Email [ajt@mit.edu](mailto:ajt@mit.edu)

1. (10 marks) Show that

$$u''(x) \approx \frac{u(x+2h) - 2u(x+h) + u(x)}{h^2} + \mathcal{O}(h).$$

2. (10 marks) The 2nd order differential equation  $-u''(x) = f(x)$ ,  $u'(0) = 0$ ,  $u'(1) = 0$  has the following  $(n+1) \times (n+1)$  matrix discretization:

$$\underbrace{\begin{pmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & & \ddots & \ddots & \ddots \\ & & & -1 & 2 & -1 \\ & & & & -1 & 1 \end{pmatrix}}_{=B_{n+1}} \begin{pmatrix} u(0) \\ u(h) \\ \vdots \\ u((n-1)h) \\ u(1) \end{pmatrix} = \begin{pmatrix} h^2 f(0) \\ h^2 f(h) \\ \vdots \\ h^2 f((n-1)h) \\ h^2 f(1) \end{pmatrix}$$

where  $h = 1/(n+1)$ . Perform elimination on  $B_4$ . Is  $B_4$  invertible? Show that  $B_{n+1}$  is not invertible for  $n \geq 4$ . Show that the differential equation does not have a unique solution.

3. (10 marks) Find examples of  $2 \times 2$  matrices such that:  
 (a)  $AB \neq BA$ ,  
 (b)  $A^2 = -I$ , where  $A$  has real entries and  $I$  is the identity matrix,  
 (c)  $B^2 = 0$ , where  $B$  has no zero entries.
4. (10 marks) Can you find a formula for the  $(i, j)$  entry of  $K_n^{-1}$ , where

$$K_n = \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & & \ddots & \ddots & \ddots \\ & & & -1 & 2 & -1 \\ & & & & -1 & 2 \end{pmatrix}?$$

(You may wish to use the MATLAB command `inv` to make a guess.) Use your formula to write down the solution  $v$  to  $K_n v = (1, 0, \dots, 0)^T$ . Which 2nd order differential equation did you just solve?