Quiz 2

~~(a)~~ Top left truss:

(i) Size(A) = 3 × 4

(ii) No, There exists a $\underline{u}$ such that $Au = 0$ (3 eq's 4 unknowns)

(iii) 1 mechanism, 0 rigid body

Top right truss:

(i) size(A) = 6 × 8

(ii) Unstable, There exists a $\underline{u}$ such that $Au = 0$ (6 eq's 8 unknowns

(iii) 1 mechanism, 1 rigid body

Bottom left truss

(i) size(A) = 6 × 6

(ii) ~~Stable. A is invertible~~ Unstable A has a zero row.

(iii) 1 mechanisms, 0 rigid body.

Bottom right truss:

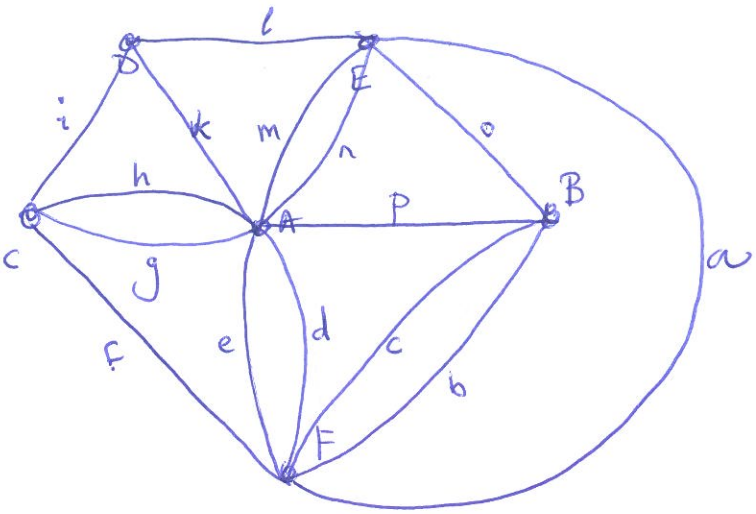
(i) size(A) = 7 × 6

(ii) Stable

(iii) None

(b) Top-left truss: $A = 3 \times 4 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

2 a)



(b) ~~A~~ WALK = SEQUENCE OF EDGES THAT CAN BE TRACED WITHOUT TAKING PEN OFF PAPER

TRAIL = A WALK THAT DOES REPEAT EDGES

EULER'S TRAIL = A TRAIL THAT INCLUDES EVERY EDGE.

(c)

LAND MASS	DEGREE
A	8
B	4
C	4
D	3
E	5
F	6

EULER'S TRAIL : (must start at D or E)

l - a - b - o - n - d - f - g - h - i - k -

e - c - p - m

[If one starts and ends at D or E, anything will work]

d) Make all vertices have an even degree.

Destroy bridge: ~~l~~ l. (between D − E)

(e) <u>NOW:</u>

LAND MASS	DEGREE
A	8
B	4
C	3
D	3
E	5
F	5

T_1 : i (single ~~e~~ edge between C − D).

T_2 : a − b − c − d − p − o − n − m − l − k − h − g − e

[any ~~tra~~ trail starting and finishing at E − F will work]

[COULD ALSO HAVE T_1 from E − D or ~~C − D~~, C − F ...]

MANY CHOICES HERE.

We need there to be at most 4 vertices of odd degree. [<u>NOTE</u>: Could also have 0 odd degree vertices OR have 2 odd degree vertices]

3a)

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

(bi) $A\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$

(ii) LOOPS: $A^T\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $A^T\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $A^T\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$

(iii) Current balancing law around the loop 2-4-6-5-3-2.

(c) Current Law: $A^T w = 0$

Voltage Law: $e^T w = 0$

(d) If $y = Ax$, $A^T z = 0$, then

$$y^T z = (Ax)^T z = x^T A^T z = x^T 0 = 0$$

$\cancel{0 = A^T z = A}$

(e) ~~Let~~ $z = w$, $y = e$, $x =$ potential differences. ~~Let $e = Ax$ by definition so $A^T z = 0 \Leftrightarrow$~~

(e) Let $z = w$, $y = e$, $x =$ potential differences

By (d) $\quad e = Ax$ by definition

$$\therefore \quad \underset{\text{current law}}{A^T w = 0} \quad \Rightarrow \quad \underset{\text{voltage law}}{e^T w = 0}$$

~~Conversely, if $e^T w = 0$ then $x^T A^T w = 0$~~

4 a) The matrix A is 3×2.

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix}.$$

Thus,

$$K = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ & & s \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & s+1 \end{bmatrix}.$$

(b) ~~So~~ Solve $\begin{bmatrix} 2 & -1 \\ -1 & s+1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} mg \\ mg \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{2s+1} \begin{bmatrix} s+1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} mg \\ mg \end{bmatrix}$$

$$= mg \begin{bmatrix} \frac{s+2}{2s+1} \\ \frac{3}{2s+1} \end{bmatrix}$$

(c) $s \to \infty$, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = mg \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$

(d) $s \to 0$, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = mg \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(e) For internal forces:

$$y = CAx = \begin{bmatrix} 1 & & \\ & 1 & \\ & & s \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= mg \begin{bmatrix} \frac{s+2}{2s+1} \\ -\frac{s-1}{2s+1} \\ -\frac{3s}{2s+1} \end{bmatrix}$$

$s \Rightarrow \cancel{\infty}\, 0$: $\quad y = mg \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

$s \rightarrow \infty$ $\quad y = mg \begin{bmatrix} 1/2 \\ -1/2 \\ -3/2 \end{bmatrix}$.

NOTE: COMMON MISTAKE IS TO TAKE THE LIMIT TOO EARLY.

[CONFUSION IN LECTURE + BOOK ON $\pm$ SIGN.
FULL MARKS TO ANY $\pm$ OF THESE ANSWERS.]