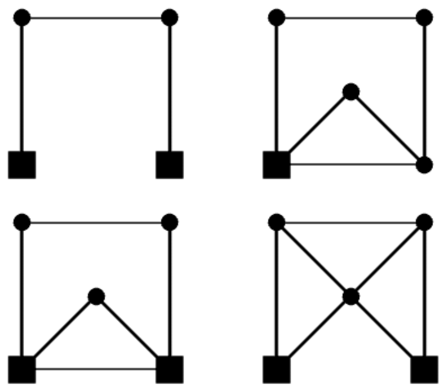
18.085 Computational Science and Engineering
Mid-term Quiz 2

Tuesday, 14th April 2015, 2:35pm to 3:55pm

There are four questions each worth 20 marks. Please write your answer to each question in the space provided. You may use your class notes, the course book, or index cards. However, you may ***not*** *use wifi-capable devices such as laptops, tablets, or cell phones.*

You may attempt as many questions as you like but ***only your three best answers will count.***

1. (20 marks) Below are four trusses.



A solid black circle is a free pin, a solid black square is a supported pin (that allows rotation), and a black line is a bar. All diagonal bars are at 45 degrees.

(a) Each truss is associated to an incidence matrix A, i.e., the matrix A in A^TCA. For each truss, answer the following questions:

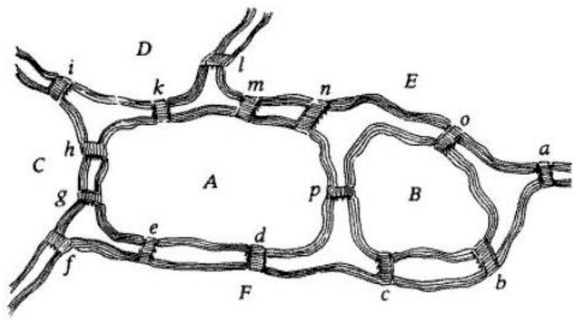
(i) What is the size of A?

(ii) Is the truss stable? Give a clear reason.

(iii) If the truss is unstable: How many unstable mechanisms? How many rigid body modes?

(b) For the first truss (top-left truss) calculate the incidence matrix A.

2. (20 marks) Consider the following map of rivers and bridges:



There are six land masses A-F and fifteen bridges a-p (j has been removed as a label).

(a) Convert the map to a graph with six vertices and fifteen edges. (Please use the same lettering on your graph as in the map.)

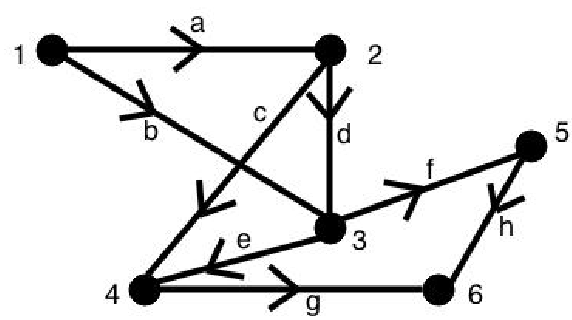
(b) Define walk, trail, and Euler's trail.

(c) Either describe an Euler's trail on the graph or explain why there is not one.

(d) Destroy one bridge so that an Euler's trail can start and finish at any land mass. Explain why it can?

(e) Restore the bridge that was destroyed in (d). Now, suppose that bridge f is in disrepair (not usable). Give two trails T_1 and T_2 so that every bridge is either in T_1 or T_2 but not both. In terms of degrees of vertices, what is the property that allows one to find T_1 and T_2?

3. (20 marks) Consider the following electrical circuit with six nodes (numbered) and eight edges (lettered):



(a) Write down the 8×6 incidence matrix A.

(b) (i) Describe the vectors x such that $Ax = 0$.
(ii) Describe the vectors w such that $A^T w = 0$.
(iii) Give the physical interpretation of $A^T w = 0$, where $w = (0, 0, 1, -1, 0, -1, 1, -1)^T$.

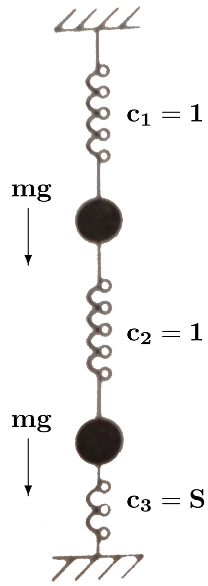
Let A be the incidence matrix, e the vector of potential differences, and w the vector of edge currents.

(c) State Kirchhoff's current and voltage law and as relations involving A, e, and w. (You may like to write down the laws in words first.)

(d) Show the following for vectors x, y, and z: If $y = Ax$ and $A^T z = 0$, then $y^T z = 0$.

(e) Explain, using the result from (d), why Kirchhoff's current implies Kirchhoff's voltage law.

4. (20 marks) Consider the following line of three springs connecting two masses:



There are two equal masses (of mass m) connected by three springs with spring constants $c_1 = 1$, $c_2 = 1$, and $c_3 = S$. Spring 1 is fixed at the top and spring 3 at the bottom. Gravity exerts an external force of mg downwards.

(a) Calculate the stiffness matrix $K = A^T CA$.

(b) Solve $Kx = f$ for the displacements.

(c) If the third spring constant $S \to \infty$, what are the limiting values of x?

(d) If the third spring constant $S \to 0$, what are the limiting values of x?

(e) For both $S \to 0$ and $S \to \infty$, what are the limiting values of the internal spring forces?